8 Quantum Interference: Wave–Particle Duality

The double-slit experiment, first carried out in 1802 by Thomas Young, played a crucial role in establishing the wave nature of light. This was in contrast to Newton's postulate that light consisted of small corpuscles. In the first quarter of the twentieth century, the concept of wave–particle duality firmly took root, motivating a deeper understating of the double-slit experiment, particularly for incident electrons instead of light beams. The experimental observation that incident electrons yield a similar interference pattern as that formed by light waves was a shocking result. Richard Feynman remarked in his famous Feynman Lectures that Young's double-slit experiment with electrons contains the deepest mystery of quantum mechanics. The only way the experimental results could be explained is via a wavefunction description of electrons. But the mystery does not stop there. If, in the same experiment, one can acquire the information about the path the electrons followed, the interference fringes disappear. This is the essence of the wave–particle duality.

Young's double-slit experiment was also at the center of the first of several debates between Albert Einstein and Niels Bohr on the foundations of quantum mechanics. Einstein came up with arguments that challenged Bohr's principle of complementarity by suggesting a clever scheme in which one can have both the wave and particle aspects exhibited in the same experiment. Bohr successfully defended the principle of complementarity by invoking Heisenberg's uncertainty relation.

The wave–particle aspect as embodied in the double-slit experiment has continued to lead to highly counterintuitive notions of delayed choice and quantum eraser effects showing how the availability or erasure of information generated in the past can affect how we interpret the data in the present. All these topics are discussed in the following sections of this chapter.

8.1 Young's Double-slit Experiment for Electrons

In Chapter 4, we discussed in great detail how, when a light beam passes through two slits, it can generate an interference pattern, a pattern of bright fringes separated by dark fringes, on a screen, as shown in Fig. 8.1*a*. The bright fringes are located at those points on the screen where the path difference between the light waves from the two slits is zero or an integral multiple of the wavelength λ, thus leading to constructive interference. The dark fringes are, on the other hand, located at those points where the path difference is equal to $(n + 1/2)\,\lambda$ (with n being an integer), leading to destructive interference.

Quantum Mechanics for Beginners: With Applications to Quantum Communication and Quantum Computing. M. Suhail Zubairy.
 Published in 2020 by Oxford University Press. DOI: 10.1093/oso/9780198854227.001.0001

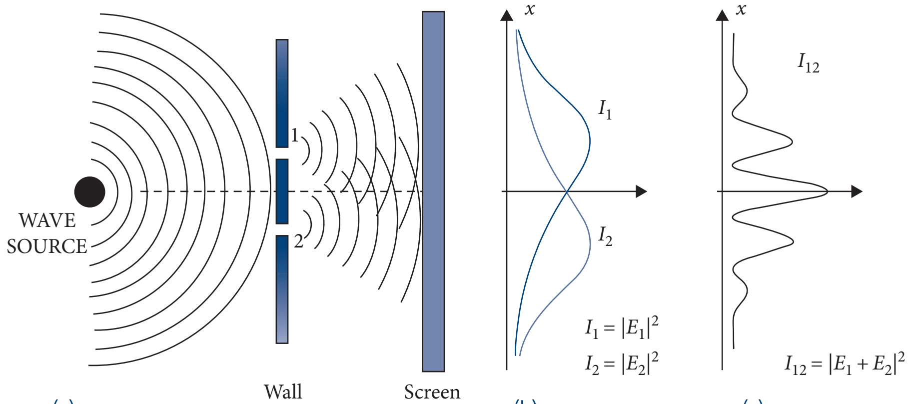

Fig. 8.1 Young's double-slit experiment with waves.

Central to this description is the wave nature of light. Light from a slit incident on the screen is described by an electric field of amplitude E. The complex amplitude of light from slit 1 is given by $E_1 = |E_1| \exp(i\delta_1)$ and from slit 2 is given by $E_2 = |E_2| \exp(i\delta_2)$. The measured intensity is given by $I = |E|^2$.

Thus, the intensity of light at the screen, when slit 2 is blocked and light can only pass through slit 1, is given by

$$I_1 = |E_1|^2 \tag{8.1}$$

and is shown in Fig. 8.1*b* by the curve I_1. Similarly when slit 1 is covered, light passes through slit 2 only and the light intensity at the screen is given by

$$I_2 = |E_2|^2 \tag{8.2}$$

and is shown by the curve I_2 in Fig. 8.1*b*.

When both slits are open, the total amplitude of light on the screen is $E_1 + E_2$ and the intensity of light is given by

$$I_{12} = |E_1 + E_2|^2. \tag{8.3}$$

Thus

$$I_{12} \neq I_1 + I_2. \tag{8.4}$$

Instead we have

$$\begin{aligned} I_{12} &= I_1 + I_2 + \left(E_1^* E_2 + E_1 E_2^*\right) \\ &= I_1 + I_2 + 2|E_1||E_2| \cos\delta \\ &= I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\delta, \end{aligned} \tag{8.5}$$

where $\delta = \delta_1 - \delta_2$ is the phase difference between the fields E_1 and E_2. The last term in the bracket is responsible for the interference. The intensity pattern on the screen is depicted by the curve I_{12} in Fig. 8.1*c*.

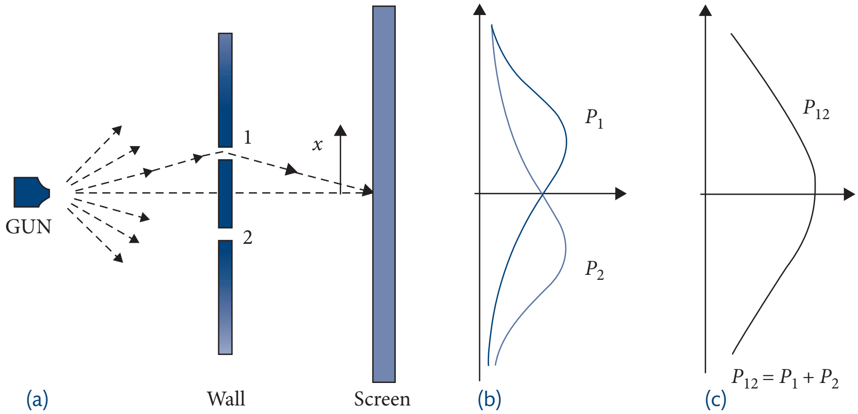

Fig. 8.2 Young's double-slit experiment with bullets.

Next we consider the double-slit experiment with particles like bullets as shown in Fig. 8.2. Here a gun is a source of bullets, sent in the forward direction, which are spread over a wide angle. These bullets can pass through holes 1 and 2 in a wall and hit a screen where they are detected. Unlike the light waves, there is no interference in this case. What we observe is the following.

When hole 2 is covered, bullets pass only through slit 1. The probability of a single bullet hitting the screen at a location x is given by P_1. This is shown by the curve marked P_1. The maximum of P_1 occurs at the value of x which is on a straight line with the gun and slit 1. When a large number of bullets are incident on the screen, their distribution (fraction of the total number of bullets hitting the screen) is given by the curve marked P_1. This curve is identical to the curve I_1 for the waves in Fig. 8.1b. When hole 1 is closed, bullets can only pass through hole 2 and we get the symmetric curve for the distribution P_2. When both holes are open, the bullets can pass through hole 1 or they can pass through hole 2 and the resulting distribution of the bullets on the screen is

$$P_{12} = P_1 + P_2. \tag{8.6}$$

The probabilities just add together. The effect with both holes open is the sum of the effects with each hole open alone. We call this result an observation of "*no interference.*" An important point to note here is that, for each bullet detected on the screen, we know (at least in principle) which hole it came from, i.e., we have the "*which-path*" information for each bullet. Indeed we can determine the full trajectory of each bullet from the point it leaves the gun and hits the screen.

So far, we have considered Young's double-slit experiment with waves and with bullets. In case of waves, the field amplitudes add and we find interference. However, when we repeat the same experiment with bullets, the probabilities add up and we find no interference.

What about Young's double-slit experiment with electrons? Do electrons behave like bullets or do they behave like waves?

We consider electrons being emitted by an electron gun. This beam of electrons passes through a wall with two slits as shown in Fig. 8.3a. The set-up is identical to the set-up for the double-slit experiment for bullets. But do we get the same result as those for the bullets?

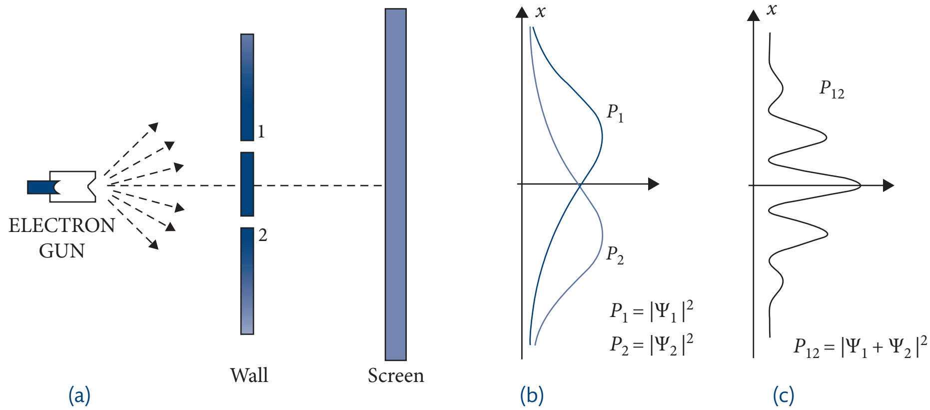

Fig. 8.3 Young's double-slit experiment with electrons.

When slit 2 is closed, electrons can only pass through slit 1. The probability of a single electron to hit the screen at location x is P_1 which is shown in Fig. 8.3*b*. The similar symmetric curve P_2 is obtained when slit 1 is closed and the electron can pass through slit 2 only. These curves are identical to the corresponding curves when the bullets are incident on the screen and also identical to the intensity distribution when a beam of light is incident.

But what happens, when both slits are open? Do electrons behave like particles as bullets do or they behave like waves as a light beam behaves? The results are shown in Fig. 8.4. Here we see the build-up of electrons on the screen. For 100 electrons, the distribution of the detected electrons on the screen appears to be random. After about 1000 electrons are detected, the distribution on the screen seems to have some regions with a dense distribution compared to other regions. But still it is difficult to conclude anything regarding the particle or wave behavior of the electrons.

After 10 000 electrons are detected on the screen, there is an unmistakable interference pattern with bright fringes, separated by dark fringes. The individual electrons are detected one by one, but instead of giving a pattern that is similar to that corresponding to bullets, we find that the electrons are detected in some regions and not in others. This is a stunning result. How do the electrons know where to hit the screen such that we see an interference pattern emerging after a large number of electrons hit the screen?

This experiment was proposed by Richard Feynman in his famous Feynman Lectures in 1965 in these words:

> We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics.

He, however, claimed that the experiment is too difficult to carry out and may never be done. What Feynman apparently did not know was that a double-slit experiment with electrons had already been done by Claus Jönsson in 1961.

The situation becomes more mysterious when a slight variation of this experiment gives us a completely different outcome.

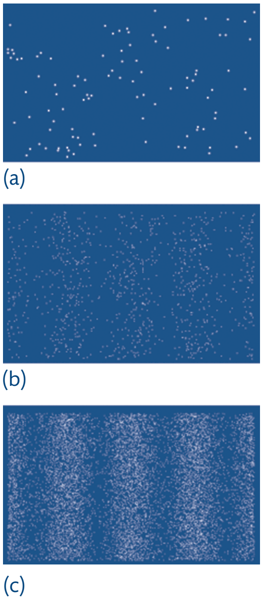

Fig. 8.4 The outcome of the Young's double-slit experiment with (*a*) 100 electrons, (*b*) 1000 electrons, and (*c*) 10 000 electrons.

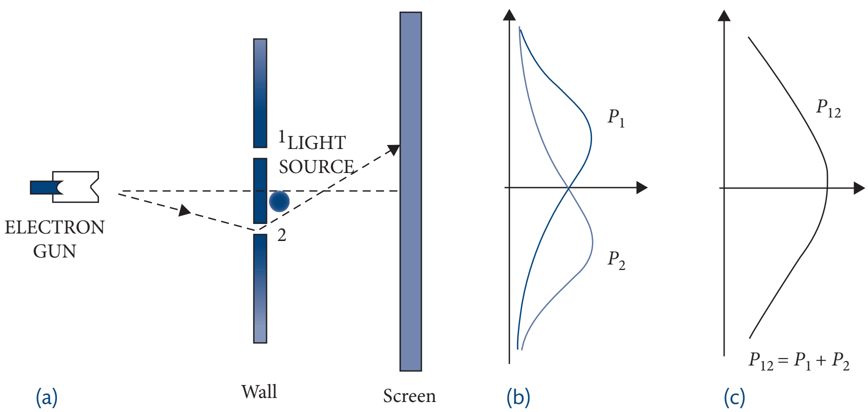

Fig. 8.5 Young's double-slit experiment with which-path information.

Let us place a source of light between the two slits as shown in Fig. 8.5. When an electron passes the slits, light scatters from the electron and provides us the which-path information. In this case, the interference disappears and the result is depicted in Fig. 8.5*c*, which is identical to the result obtained for the double-slit experiment with bullets. This is in contrast to the experiment depicted in Fig. 8.3, where we had a lack of knowledge about the path each individual electron took. This lack of which-path knowledge seems to be responsible for interference.

Thus if we "look" at *which path* each electron followed, the interference disappears and we get the same distribution on the screen as for particles. So either we get interference when we have *no which-path* information or we lose interference when we have the *which-path* information.

No classical explanation can describe these observations. We can reconcile these observations only with the fundamental principles of quantum mechanics discussed in Chapter 5.

We describe the electron, not as a particle traveling in a well-defined trajectory, but by a wavefunction $\psi(\mathbf{r})$ which is a complex function of position. At any point R on the screen, there are two contributions for the same electron coming from the two slits, $\psi_1(R)$ and $\psi_2(R)$.

When slit 2 is closed, the total wavefunction at the position R is $\psi_1(R)$ and the probability of finding the electron is

$$P_1 = |\psi_1|^2 \tag{8.7}$$

Similarly, when slit 1 is closed, the total wavefunction at the position R is $\psi_2(R)$ and the probability of finding the electron is

$$P_2 = |\psi_2|^2 \tag{8.8}$$

When both slits are open, the total wavefunction of the electron at the position R is

$$\psi(R) = \psi_1(R) + \psi_2(R) \tag{8.9}$$

and the probability of finding the electron is

$$\begin{aligned} P_{12} &= |\psi_1 + \psi_2|^2 \\ &= |\psi_1|^2 + |\psi_2|^2 + (\psi_1^* \psi_2 + \psi_1 \psi_2^*) = |\psi_1|^2 + |\psi_2|^2 + 2|\psi_1||\psi_2| \cos\theta. \end{aligned} \tag{8.10}$$

Here $\psi_1 = |\psi_1|\ exp(i\theta_1), \psi_2 = |\psi_2|\ exp(i\theta_2)$, and $\theta = \theta_1 - \theta_2$. The angle θ depends on the location on the screen. The last term is the interference term which, depending on θ, can become equal and opposite to $|\psi_1|^2 + |\psi_2|^2$ at certain locations, giving us a zero probability of finding the electron at those locations and is responsible for the interference. The wavefunctions ψ_1 and ψ_2 seem to play the same role as the complex fields E_1 and E_2 in the case of Young's double-slit experiment with waves. However there is one crucial difference: The quantities $I_1 = |E_1(R)|^2$ and $I_2 = |E_2(R)|^2$ are the intensities of the light coming to the point R from slits 1 and 2 whereas the quantities $|\psi_1(R)|^2$ and $|\psi_2(R)|^2$ are the probabilities that the electron coming from slits 1 and 2 hits the screen at the point R, respectively.

If an experiment is performed which is capable of determining whether the electrons passed through slit 1 or slit 2, the probability of finding the electron at a point R on the screen is the sum of the probabilities for each alternative,

$$P_{12} = P_1 + P_2, \tag{8.11}$$

and the interference is lost.

This concept of wave–particle duality has been a source of intense discussion since the earliest days of quantum mechanics. How the same electron can behave like a wave in one situation and a particle in another is quite mysterious. Wave–particle duality was the subject of a fierce debate between Albert Einstein and Niels Bohr, as we discuss in the next section.

Richard Feynman expresses his amazement at these incredible results in these words:

> One might still like to ask: "How does it work? What is the machinery behind the law?" No one has found any machinery behind the law. No one can "explain" any more than we have just "explained." No one will give you any deeper representation of the situation. We have no ideas about a more basic mechanism from which these results can be deduced.

Here we discussed the experiment with electrons. The same can be said about a similar experiment with light. If we treat the light beam in a Young's double-slit experiment as consisting of a large number of photons, the situation is similar to the interference experiment with electrons. The reason we get interference of a light beam in a double-slit experiment is due to the lack of which-path information for each photon. If somehow we are able to get which-path information for each photon, the interference disappears.

This is the essence of the wave–particle duality or Bohr's principle of complementarity: Electrons and photons can behave like waves when we have no which-path information and they behave like particles when we have the which-path information.

8.2 Einstein–Bohr Debate on Complementarity

In 1927 Niels Bohr introduced the principle of complementarity, which can be stated as follows: In any quantum mechanical experiment, certain physical concepts are complementary. If the experiment clearly illustrates one concept the other concept will be completely obscured. As an example, if the particle nature of an object is exhibited in an experiment then the wave nature will be completely obscured. Thus, in the double-slit experiment, we can either have the which-path information or the existence of an interference pattern. According to Bohr's principle of complementarity, they can never be observed at the same time, in the same experiment.

Einstein, however, came up with a clever scheme such that we can have both which-path information (particle nature) and interference (wave nature) in the same experiment—violating Bohr's principle of complementarity.

In Einstein's proposed experiment a wall with two slits is placed on rollers so that it can move freely in the vertical direction as shown in Fig. 8.6. This is an example of what we call a *thought experiment* or a *gedanken experiment*. In other words, we do not *actually* conduct the experiment, we use only our imagination and reasoning instead. An electron gun shoots electrons towards the wall where they can pass through the two slits and then onto the back screen to create the interference pattern. The electrons have momentum in the forward direction. However the electron beam is spread and can have small momentum components in both $+x$ and $-x$ directions. For example, the electrons passing through slit 1 should have a momentum component along the x-axis equal to p_1 and those passing through slit 2 should have the x-momentum component equal to p_2. After passing through the slits, there is a momentum change in the electrons. For those passing through slit 1, if the final momentum in the x-direction is p'_1, the momentum change of that electron is $\delta p_1 = p'_1 - p_1$. Similarly for the electrons passing through slit 2, the momentum change is $\delta p_2 = p'_2 - p_2$.

Einstein argued that if the wall is on rollers then, by the law of conservation of momentum, it should recoil with a momentum equal to the change in momentum of the electron and in

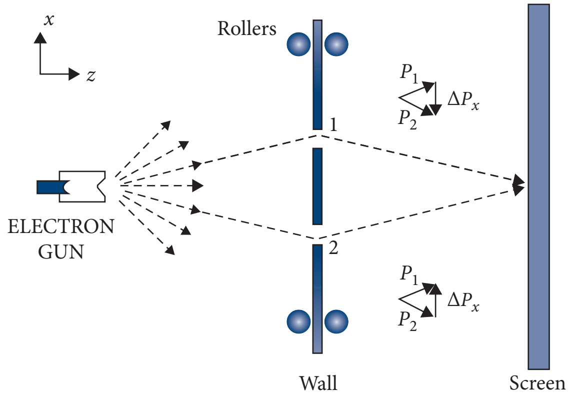

Fig. 8.6 Einstein's *gedanken* experiment. Electrons pass through a narrow slit through a wall that can freely move on a roller. The momentum transfer to the wall provides the which-path information. At the same time electrons from the two slits can form an interference pattern on the screen.

the opposite direction. Thus electrons that pass through the upper slit (slit 1) should impart a momentum equal to $-\delta p_1$ to the wall. If the electron is deflected in the downward direction as shown in Fig. 8.6, then the momentum of the wall should be in the upward direction. Similarly, the electrons passing through the lower slit (slit 2) and deflected in the upper direction will give a downward kick to the wall. Therefore, for every position of the detector on the screen, the momentum received by the wall will have a different value for a traversal via slit 1 than for a traversal via slit 2. Since an electron has a very small mass, the momentum change is very tiny and it may be difficult to measure the momentum change of the wall. However, no matter how small this momentum is, it should in principle be detectable. So without disturbing the electrons *at all*, but just by watching the *wall*, we can tell which path the electron used.

Einstein then argued that, after passage through the slits, the undisturbed electrons can proceed to the screen and give the interference pattern as before. However, we can get the information about which slit the electrons passed through by measuring the momentum of the wall after each electron has passed through. Thus we have both the 'which-path' information and the interference. This is in contradiction to Bohr's principle of complementarity.

This was a forceful argument against the foundational principle of quantum mechanics and Bohr had to respond to it immediately. Leon Rosenfeld records the encounter in his book *Fundamental Problems in Elementary Particle Physics* (Proceedings of the Fourteenth Solvay Conference, Interscience, New York, p. 232) in the following words:

> ... Einstein thought he had found a counterexample to the uncertainty principle. It was quite a shock for Bohr ... he did not see the solution at once. During the whole evening he was extremely unhappy, going from one to the other and trying to persuade them that it couldn't be true, that it would be the end of physics if Einstein were right; but he couldn't produce any refutation. I shall never forget the vision of the two antagonists leaving the club [of the Fondation Universitaire]: Einstein a tall majestic figure, walking quietly, with a somewhat ironical smile, and Bohr trotting near him, very excited ... The next morning came Bohr's triumph.

Bohr invoked the Heisenberg uncertainty relation to refute Einstein's argument and saved the principle of complementarity.

According to the Heisenberg uncertainty relation, if we determine the x-component of its momentum with an uncertainty Δp, we cannot, at the same time, know its x-position more accurately than $\Delta x = \hbar/2\Delta p$ (Section 7.4). In Einstein's argument, it is necessary to know the momentum of the wall before the electron passes through it sufficiently precisely. This is required as we need to know the change in the momentum of the wall in the x-direction after the electron has passed in order to obtain the which-path information. However, according to the Heisenberg uncertainty principle, we cannot know the position of the wall in the x-direction with arbitrary accuracy. Therefore a precise measurement of momentum means that the locations of the slits become indeterminate. The uncertainty in the location of the slits means that the electrons effectively see a blurred pair of slits. The locations where electrons hit the screen consequently become random and the center of the interference pattern has a different location for each electron, thus wiping out the interference pattern. This shows that the which-path information in the Young's double-slit experiment smears the interference pattern.

In order to quantitatively see this result, we consider a slightly different set-up as shown in Fig. 8.7. Here a beam of electrons is first sent along the z-axis through a wall with a narrow opening that selects only those electrons moving along the z-axis. Before hitting the wall the x-component of the momentum of these electrons is zero. After passing through the slit they diffract in the x-direction. Electrons can pass through another wall at a distance L with double slits. The separation between the two slits is d. The electrons are detected on the screen another distance L away. The first wall is placed on a roller such that it can freely move in the x-direction.

The incident electrons move with a momentum

$$p_0 = \frac{h}{\lambda} \tag{8.12}$$

along the z-axis. Here λ is the de Broglie wavelength of the electrons. After passing the first wall, they acquire momentum in the x-direction. Since these electrons pass through the two

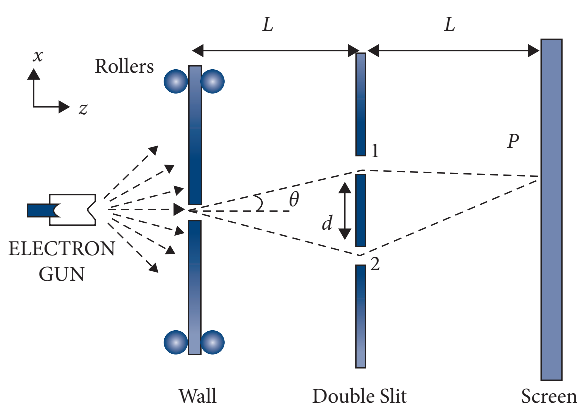

Fig. 8.7 An analysis of Einstein's *gedanken* experiment. Electrons pass through a wall giving it a push in the upward or the downward direction depending on whether the incoming electron is scattered in the downward or upward direction. This provides the which-path information. These electrons then pass the double slit and form a pattern on the screen.

slits at the distance L from the first wall and located at $x = \pm(d/2)$ the x-component of the momentum can range from $-p_0\sin\theta$ for electrons going below the z-axis to $+p_0\sin\theta$ for electrons going above the axis. By the conservation of momentum, the corresponding recoil momentum on the first wall therefore ranges from $+p_0\sin\theta$ for electrons going below the z-axis to $-p_0\sin\theta$ for electrons going above the axis. Thus the limit on the accuracy of measuring the recoil momentum is

$$\Delta p = +p_0 \sin\theta - \left(-p_0 \sin\theta\right) = 2p_0 \sin\theta \approx 2p_0\theta = 2\frac{h}{\lambda}\frac{d}{2L} = \frac{hd}{\lambda L}, \tag{8.13}$$

where we assume $\theta \ll 1$ and $\sin\,\theta \approx \theta$. According to the Heisenberg uncertainty relation, the minimum uncertainty in the position of the source slit is

$$\Delta x \approx \frac{h}{\Delta p} = \frac{\lambda L}{d}, \tag{8.14}$$

where we substituted for Δp from Eq. (8.13). Thus, if the electron momentum in the x-direction is known with sufficient accuracy to find the *which-slit* information, the location of the slit in the first wall is uncertain by an amount given by Eq. (8.14). This leads to a corresponding uncertainty in the location where the electron hits the screen. We recall that the fringe spacing in the double slit experiment is $\lambda L/d$ (Eq. (4.54). The resulting pattern on the screen becomes blurred to the extent that the interference pattern is lost. This clearly shows that the which-path information leads to the disappearance of the interference pattern—Bohr's principle of complementarity is saved, thanks to the Heisenberg uncertainty relation.

Richard Feynman, in his Lectures, describes the role of the uncertainty relation in keeping the foundations of quantum mechanics secure in the following words:

> The uncertainty principle "protects" quantum mechanics. Heisenberg recognized that if it were possible to measure the momentum and the position simultaneously with a greater accuracy, then quantum mechanics would collapse. So he proposed that it must be impossible. Then people sat down and tried to figure out ways of doing it, and nobody could figure out a way to measure the position and the momentum of anything—a screen, an electron, a billiard ball, anything—with any greater accuracy. Quantum mechanics maintains its perilous but still correct existence.

8.3 **Delayed Choice**

In the Young's double-slit experiment, whether we get the interference fringes or not depends on whether we have no which-path information or we have the which-path information. Thus a photon behaves like a wave or a particle depending upon what kind of an experiment we decide to do. If we decide not to look at the photon when it is passing through the slits, it behaves like a wave. However, if we decide to find which slit the photon goes through, it behaves like a particle. This wave–particle duality is very mysterious and it becomes even more so when we try to address the question whether the photon knew in advance what behavior it should exhibit. This question was addressed by John Wheeler in his "delayed choice" *gedanken* experiment.

In Wheeler's *gedanken* experiment, photons are generated by cosmic objects like quasars. They are split into two paths with the galaxy acting as a gravitational lens as shown in Fig. 8.8. Photons can follow either path, left of the galaxy or right of the galaxy. After having traveled a distance of billions of miles the photons arrive at earth where we detect these photons in one of the two different experimental set-ups.

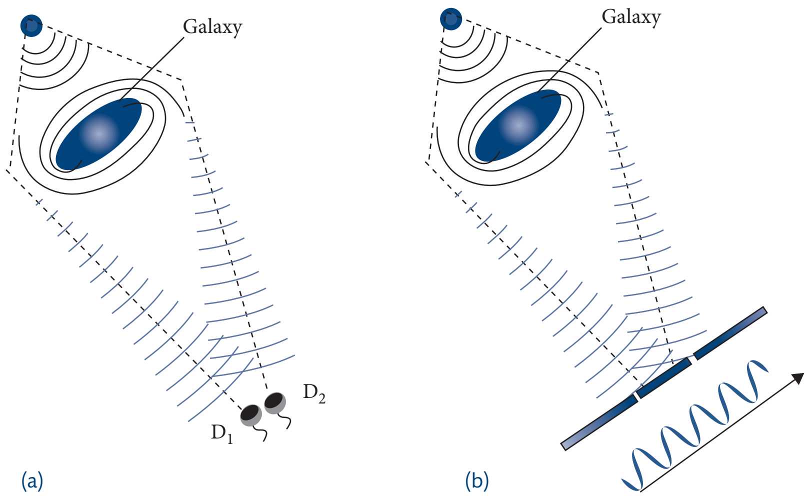

Fig. 8.8 Wheeler's delayed choice experiment. Light that left a quasar millions of years ago can be made to act like a particle or a wave depending on our choice of the experimental set-up.

In the first set-up, we place two detectors D_1 and D_2. The detector D_1 clicks if the photon followed the left path and the detector D_2 clicks if the photon followed the right path. Thus a click at either D_1 or D_2 provides the which-path information. For example, for a click at D_1, we can conclude that the photon was in the left path all along for all those billions of years. Similarly, for a click at D_2, we can conclude that the photon was in the right path all along.

The other possibility is to pass them through the two slits in a Young's double-slit experiment and get an interference pattern. We can then conclude that the photons behaved like waves and they went through both ways around the galaxy.

Therefore, in the first case the photons appear to pass through only one side of the galaxy and behave like particles and in the latter case they behave like waves and go through both ways around the galaxy. The paradoxical situation is that it depends on the experimenter's "delayed choice" whether the photon generated billions of years ago behaves like a particle or a wave. Until the experiment is done, we cannot say whether the photon will behave as a particle or as a wave.

8.4 **Quantum Eraser**

An even more counterintuitive aspect of wave–particle duality is the notion of the "quantum eraser" introduced by Marlan Scully and Kai Drühl in 1982. In the Young's double-slit experiment, we get an interference pattern if we have no knowledge about which slit the photon went through. However if we somehow obtain the which-path information then the interference is lost. Scully posed the question: Is it possible to "erase" the which-path information and recover the interference pattern *after* the photon has passed through the slits and is detected on the screen? The quantum eraser brings out the counterintuitive aspects related to time in the quantum mechanical domain.

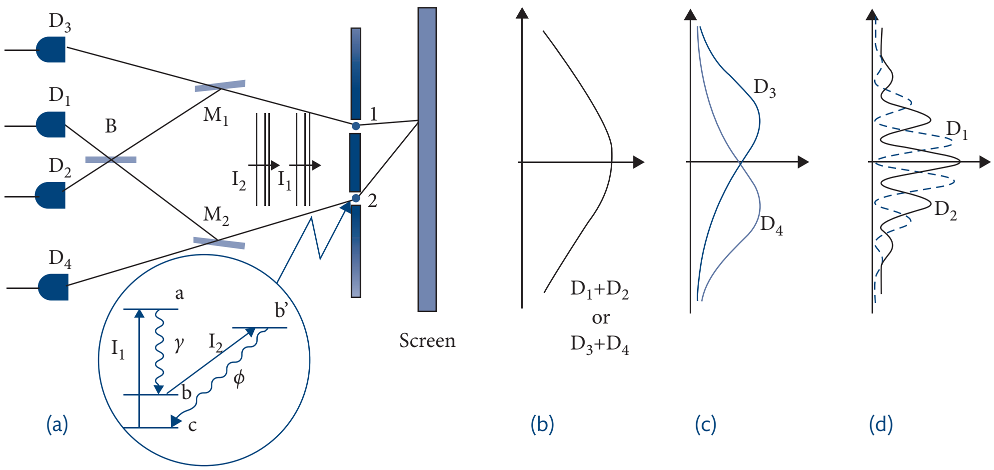

Fig. 8.9 Schematics of the quantum eraser experiment. (*a*) The single-photon pulses l_1 and l_2 incident on two atoms help to generate two photons γ and ϕ by either atoms at site 1 or the atom at site 2. The γ photons proceed to the screen and the ϕ photon to the left. (*b*) The distribution of γ photons when no detection is made at detectors D_1, D_2, D_3, or D_4. This distribution is also obtained when either both mirrors M_1 and M_2 are removed and the clicks are at detectors D_3 and D_4 or the mirrors M_1 and M_2 are in place and the clicks are at detectors D_1 and D_2. (*c*) The distributions of the γ photons for clicks at D_3 and D_4. The which-path information destroys the interference. (*d*) The distributions of the γ photons for clicks at D_1 and D_2. In this case we do not have the which-path information and interference is obtained.

We present a simple description of the quantum eraser as depicted in Fig. 8.9. Instead of two slits, we consider the scattering of light from two atoms on the screen.

The two atoms are placed at sites 1 and 2. Each atom is of the type shown in the inset of Fig. 8.9. There are four atomic levels a, b, b', and c and the atoms are initially in level c. These atomic levels are of the type we discussed for the hydrogen atom in Section 6.5. The atom can absorb a photon and make a jump from a lower level to a higher level if the energy difference between the two levels is the same as the energy of the incident photon. Similarly, an atom in the excited state can jump to the lower level and emit a photon whose energy (and frequency) matches the level spacing. These atoms are excited by pulses l_1 and l_2 which carry just enough energy to excite only one atom from level c to a and from level b to b', respectively.

The photon pulse l_1 tuned to c-a transition excites one atom (we do not know which one) to level a. The other atom remains in level c. The excited atom emits a photon by making a jump from level a to level b. We call such a photon a γ photon. The photon pulse l_2 excites the atom from level b to b'. The atom finally makes a transition from level b' to level c emitting a photon we call ϕ photon. Thus, after the passage of the pulses l_1 and l_2, one of the atoms (we do not know which one) has generated two photons, γ and ϕ and both atoms are found in the ground state c after the scattering process is complete.

We repeat this scattering process a large number of times. We consider only those instances where the γ photon proceeds to the right to the screen and the ϕ photon proceeds to the left to the mirrors M_1 and M_2. The γ photons are collected on the screen as in the usual double-slit experiment. The ϕ photon is detected by one of the detectors D_1, D_2, D_3, or D_4 after passing through the optical set-up consisting of the mirrors M_1, M_2, and the beam splitter B. The role of the beam splitter B is to let the photon get transmitted or get reflected with equal probability.

For example, a photon reflected from the mirror M_1 can either get reflected through B and be detected at D_1 or be transmitted and detected at D_2, with equal probability. A detailed analysis of a beam splitter for a single photon is given in Section 9.4. The γ photons from atom 1 or atom 2 play the same role as the light passing through the two slits in the Young's double-slit experiment. The ϕ photons can be employed to manipulate the which-path information as described below.

This experiment yields a distribution of γ photons on the screen as shown in Fig. 8.9. But what about the appearance and disappearance of interference fringes discussed above? For this purpose we look at the ϕ photons that proceed to the left.

The ϕ photon, if emitted by atom 1, proceeds to mirror M_1 and, if emitted by atom 2, to mirror M_2. The distance between the screen (with two atoms) and the mirrors M_1 and M_2 is assumed to be much larger than the distance between the atoms and the screen where the γ photons are detected.

For each ϕ photon, a choice is made: either both mirrors M_1 and M_2 are removed OR they are kept in place. In the case where the mirrors M_1 and M_2 are removed the photon proceeds unhindered and there is a click at either detector D_3 or D_4. On the other hand, if the mirrors M_1 and M_2 are in place, there is a click either at detector D_1 or D_2.

For each detection of a γ photon on the screen, we thus have four possibilities for the detection of the corresponding ϕ photon: It can be detected at detectors D_1 or D_2 or at detectors D_3 or D_4 depending on whether the mirrors M_1 and M_2 are in place or they are removed. Let us examine these cases.

First we consider the case when a decision is made to remove both mirrors M_1 and M_2. In this case there is a click either at D_3 or D_4.

If the ϕ photon is detected at D_3, there is only one path possible, namely $1D_3$. The ϕ photon must have come from atom 1. We thus have the information about the atom that generated the ϕ photon. The corresponding γ photon must have been generated by atom 1 as well and we acquire the which-path information for the γ photon on the screen.

Following the same reasoning, we conclude that, if the ϕ photon is detected at D_4, it must have come from atom 2. The corresponding γ photon must have been generated by atom 1 as well and, again, we acquire the which-path information for the γ photon on the screen.

Next we consider the case when a decision is made to keep both mirrors M_1 and M_2. In this case there is a click either at D_1 or D_2.

If the ϕ photon is detected at D_1, there is an equal probability that it may have come from the atom located at 1 following the path $1M_1BD_1$ or it may have come from the atom located at 2 following the path $2M_2BD_1$. Thus we have erased the information about which atom scattered the ϕ photon and there is no which-path information available for the corresponding γ photon.

The same can be said about the ϕ photon detected at D_2. There is an equal probability that it may have come from the atom located at 1 following the path $1M_1BD_2$ or it may have come from the atom located at 2 following the path $2M_2BD_2$. There is, however, a phase shift of π, as we get two reflections in case 1 and one reflection and one transmission in case 2.[1]

[1] The π phase shift is understood using the properties of the beam splitter that we formally derive in Sections 9.3 and 9.4.

After this experiment is done a large number of times, we shall have ϕ photons detected each at detectors D_1, D_2, D_3, or D_4. The spatial distribution for all the collected γ photons in the absence of any sorting is given in Fig. 8.9*b*. Next we do a sorting process. We separate out all the events where the ϕ photons are detected at detectors D_1, D_2, D_3, and D_4. For these four groups of events, we locate the positions of the detected γ photons on the screen.

And now comes the key result! For the events corresponding to the detection of ϕ photons at detectors D_3 and D_4, the pattern obtained by the γ photons on the screen is the same as we would expect if these photons had scattered from atoms at sites 1 and 2, respectively. This is shown in Fig. 8.9*c*. That is, there are no interference fringes as would be expected when we have which-path information available. On the contrary, we obtain phase-shifted interference fringes for those events where the ϕ photons are detected at D_1 and D_2. This is shown in Fig, 8.9*d*. For this set of data there is no which-path information available for the corresponding γ photons.

Mathematically we can understand the essential results of the Scully–Drühl quantum eraser by first realizing that the photon state emitted by the atoms located at sites 1 and 2 is given by

$$\Psi = \frac{1}{\sqrt{2}}\left(\psi_{\gamma_1}\psi_{\phi_1} + \psi_{\gamma_2}\psi_{\phi_2}\right), \tag{8.15}$$

i.e., either the photon pair γ_1, ϕ_1 is emitted by the atom located at site 1 or pair γ_2, ϕ_2 is emitted by the atom located at site 2. Thus if the ϕ photon is detected by D_3, the quantum state reduces to ψ_{γ_1}. A similar result is obtained for the ϕ photon detection by D_4. This is the situation when the which-path information is available and the sorted data yields no interference fringes.

The physics behind the retrieval of the fringes is made clear by rewriting the state Ψ as[2]

$$\Psi = \frac{1}{2}\left(\psi_{\gamma_1} + \psi_{\gamma_2}\right)\psi_{\phi_+} + \frac{1}{2}\left(\psi_{\gamma_1} - \psi_{\gamma_2}\right)\psi_{\phi_-}, \tag{8.16}$$

where

$$\psi_{\phi_+} = \frac{1}{\sqrt{2}}\left(\psi_{\phi_1} + \psi_{\phi_2}\right) \tag{8.17}$$

is the symmetric state of the ϕ photon at the detector D_1 after passage through the beam splitter B, and

$$\psi_{\phi_-} = \frac{1}{\sqrt{2}}\left(\psi_{\phi_1} - \psi_{\phi_2}\right), \tag{8.18}$$

is the antisymmetric state of the ϕ photon at the detector D_2 after passage through the beam splitter B. Thus a click at detectors D_1 or D_2 reduces the state of a γ photon to

$$\psi_{\gamma_+} = \frac{1}{\sqrt{2}}\left(\psi_{\gamma_1} + \psi_{\gamma_2}\right) \tag{8.19}$$

[2] Here again the symmetric and antisymmetric states ψ_{ϕ_+} and ψ_{ϕ_-} are obtained at the detectors D_1 and D_2, respectively, by using the properties of the beam splitter that we derive in Sections 9.3 and 9.4.

or

$$\psi_{\gamma_-} = \frac{1}{\sqrt{2}}\left(\psi_{\gamma_1} - \psi_{\gamma_2}\right), \tag{8.20}$$

respectively, leading to a retrieval of the interference fringes according to Eqs. (8.9) and (8.10).

Thus, in summary, a detection of the ϕ photon at the detectors D_3 or D_4 corresponds to the probabilities $|\psi_{\gamma_1}|^2$ or $|\psi_{\gamma_2}|^2$, respectively, of the γ photon being detected on the screen, leading to no interference. This situation is similar to the double-slit experiment with which-path information as in Fig. 8.4 and the results are depicted in Fig. 8.9*c*. However, a detection of the ϕ photon at detectors D_1 or D_2 corresponds to the probabilities $|\psi_{\gamma_1} + \psi_{\gamma_2}|^2$ or $|\psi_{\gamma_1} - \psi_{\gamma_2}|^2$, respectively, of the γ photon being detected on the screen, leading to interference in both cases due to a lack of which-path information as in Fig. 8.3, and the present result is shown in Fig. 8.9*d*.

The remarkable result is that we can place the ϕ photon detectors, D_1, D_2, D_3, and D_4, far away—very far away, such that we can make the decision whether to remove the mirrors M_1 and M_2, thus acquiring the which-path information, or to place the mirrors to lose the which-path information long after the γ photon is detected on the screen. Thus the future measurements on the ϕ photons influence the way we think about the γ photons measured today (or yesterday!). For example, we can conclude that γ photons, whose ϕ partners were successfully used to ascertain which-path information by removing the mirrors M_1 and M_2 resulting in clicks at D_3 or D_4, can be described as having (in the past) originated from site 1 or site 2. We can also conclude that γ photons, whose ϕ partners had their which-path information erased by placing the mirrors M_1 and M_2 resulting in clicks at D_1 and D_2, cannot be described as having (in the past) originated from site 1 or site 2, but must be described, in the same sense, as having come from both sites. The future helps shape the story we tell of the past. This is a highly counterintuitive and startling result. The scheme for the quantum eraser discussed above has been realized experimentally.

In his book, *The Fabric of the Cosmos*, Brian Greene sums up beautifully the counterintuitive outcome of the experimental realizations of the quantum eraser:

> These experiments are a magnificent affront to our conventional notions of space and time. Something that takes place long after and far away from something else nevertheless is vital to our description of that something else. By any classical-common sense-reckoning, that's, well, crazy. Of course, that's the point: classical reckoning is the wrong kind of reckoning to use in a quantum universe. For a few days after I learned of these experiments, I remember feeling elated. I felt I'd been given a glimpse into a veiled side of reality.

Problems

8.1 Electrons of momentum p fall normally on a pair of slits separated by a distance d. What is the distance between adjacent maxima of the interference fringe pattern formed on a screen a distance L beyond the slits? *Note: You may assume that the width of the slits is much less than the electron de Broglie wavelength.*

8.2 In an experiment performed by Jönsson in 1961, electrons were accelerated through a 50 kV potential towards two slits separated by a distance $d = 2 \times 10^{-4}$ cm, then detected on a screen $L = 35$ cm beyond the slits. Calculate the electron's de Broglie wavelength, λ, and the fringe spacing Δy. *Note: kinetic energy of electrons is equal to* eV.

8.3 In an interference experiment with electrons, we find the most intense fringe is at $y = 7.0$ cm. There are slightly weaker fringes at $y = 6.0$ cm and 8.0 cm, still weaker fringes at $y = 4.0$ cm and 10.0 cm. No electron are detected at $y < 0$ cm or $y > 14$ cm. Sketch a graph of $|\psi|^2$.

BIBLIOGRAPHY

R. P. Feynman, R. Leighton, and M. Sands, *The Feynman Lectures on Physics, Vol. IIIA* (Addison-Wesley, Reading, MA 1965).

C. Jönsson, Zeitschrift für Physik, 161, 454 (1961); translated in C. Jönsson, *Electron Diffraction at Multiple Slits*, American Journal of Physics 42, 4 (1974).

J. A. Wheeler, *The "Past" and the "Delayed-Choice Double-Slit Experiment"*, in A.R. Marlow, editor, *Mathematical Foundations of Quantum Theory*, Academic Press (1978).

M. O. Scully and K. Drühl, *Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics*, Physical Review A 25, 2208 (1982).

M. O. Scully, B.-G. Englert, and H. Walther, *Quantum optical tests of complementarity*, Nature 351,111 (1991).

Y. Aharonov and M. S. Zubairy, *Time and the quantum: Erasing the past and impacting the future*, Science 307, 875 (2005).

S. P. Walborn, M. O. Terra Cunha, S. Pádua, and C. H. Monken, *Double-slit quantum eraser*, Physical Review A 65**,** 033818 (2002).

Y.-H. Kim, R. Yu, S. P. Kulik, Y. Shih, and M. O. Scully, *Delayed "choice" quantum eraser*, Physical Review Letters 84, 1 (2000).

R. J. Scully and M. O. Scully, *The Demon and the Quantum* (John-Wiley-VCH, 2010).

M. O. Scully and M. S. Zubairy, *Quantum Optics* (Cambridge University Press, 1997).

B. Greene, *The Fabric of the Cosmos* (Alfred A. Knopf, New York 2004).