Physics 501-22

Two time Spin measurement effect on prediction

In clasical physics, one has the equations of motion (eg $F = ma$) which are valid for all motions and conditions. In order to specify the system to reflect the world we actually want to apply these laws to, one has to supply something else. That is usually called the initial conditions– eg, given F one needs to specify X and P for the particle– their values, not their functional form. Given those initial values one can calculate the position and momentum of the particle at any time.

In the case of Quantum Mechanics, the specification of the particulars of the world to which one wants to apply the solution is done by the state of the system. In the Schroedinger case, this looks very similar to classical, namely one specifies the inital condition of the wave-function. In the Heisenberg case, it is the operaters are time dependent, but one cannot specify the intial conditions of the the "value" of the operators, since the operators cannot be represented by numbers. They are not functions, they are operators. Instead one represents the conditions by the states. They are not "initial", they are states which are the same at all times.

In clasical physics, a deterministic theory, if one applies conditions at any other time than the intial time (however you want to define that) then those conditions can always be transfered to the intial time, and intial conditions via the equations of motion. All conditions are equivalent to initial conditions.

However, as Heisenberg and Einstein and Tolman pointed out, the same does not appear to be true for quantum mechanics. Is quantum mechanics valid for future conditions, or for conditions both in the future and the past?

Let us look at a specific problem. Consider a two level system (it could say be a spin 1/2 particle, or the polarization of a light beam, or the path of a photon through the two arms of an interferometer). The operators for this system will be taken as the usual Pauli matrices $\sigma_1,\ \sigma_2,\ \sigma_2$. Let us assume that the system we are interested in for this two level system has an intrinsic Hamiltonian of 0. One can of course make measurements on the system by interacting on it via some external measuring apparatus. Let us make the following measurements and place the following conditions on the system. At 9AM I measure σ_3 and find that its value is +1. At 11AM I measure σ_1 on the same system, and find that its value is +1. I now want to ask whetehr or not I can say anything about the system in the intermediate time of 10AM. Now one can make this an ontological question– is there something inherently true about the system at 10AM that those two conditions allow me to say. But instead of that lets follow Heisenberg and ask about measurements, rather than about ontology. At least measurements allows us to, for example do experiments, and thus ask the physical world how it behaves, rather than philosophising about it.

So let me phrase the question in the following way. If my graduate student came into the lab and at 10AM carried out an experiment on that same system, in which she measured the operator $\cos(\theta)\sigma_3 + \sin(theta)\sigma_1$. She asks me what are the probabilities that she found the value of +1 at 10AM?

Heisenberg's answer would appear to be something like "I have no idea. That is a question outside the ability of my theory of the world to answer."

But surely that is not correct. Let us look at two possibilities. Lets say that the student chose $\theta = 0$, and thus it was σ_3 that was measured. We know that the Hamiltonian between 9 and 10 was zero, so nothing changed. At 9 , Σ_3 was found to have value 1. Since nothing happened between 9 and 10, the vlue of σ_3 must still be 1 and the probability must be certainty that she would have measured σ_3 and found the value of 1.

Let us now say that she measured the attribute with $\theta = \pi/2$. Again, had she got the value of -1, then since nothing happened between 10 and 11, I would have had to have found the value of -1 at 11, in contrast to the value I did get, namely 1. Thus she must have gotten the value of +1 with certainty. Thus, whatever else, the value of σ_θ must have the probability of certainty of getting the value 1 at 10. Of course there exists no state for the system which would give these two results. One cannot therefor encode the conditions into the theory by a wave function. But surely I used impecable quantum reasoning to draw those conclusions.

Wave functions are not quantum mechanics. They are one way of encoding knowledge under certain circumstances into the calculation– in particular encoding information from the past (or the future) into the calculation. Wave functions are not "real"– they are not something that exists out there. They are devices for enabling one to incorporate knowledge of the world into quantum mechanics.

Lets now ask what the probability would be for getting the value of 1 had the measurement of σ_θ for arbitrary θ been carried out. I would argue as follows

By quantum mechanics, the state after 9AM would be the eigenstate of σ_3 which I can represent by $|+1\rangle$. At 10 if she had gotten the value +1 for σ_θ, that would correspond to the eigenstate of σ_θ which would be $\cos(\theta/2)|1\rangle + \sin(\theta/2)|-1\rangle$, with probability

$$P_{1_9 1_{10}} = |(\cos(\theta/2)\langle 1| + \sin(\theta/2)\langle -1|)|1\rangle|^2 = cos^2(\theta/2) \quad (1)$$

. Now given that the one got the value of +1 for σ_θ at 10, the probability of getting +1 for σ_1 at 11 would be

$$\frac{P_{1_1 0 1_1 1} = |(1}{\sqrt{2})(\langle 1| + \langle -1|)(\cos(\theta/2)|1\rangle + \sin(\theta/2)|-1\rangle)| = \frac{1}{2}(cos(\theta/2) + sin(\theta/2))^2} \quad (2)$$

Similarly the probability that at 10 she would have gotten -1 is

$$P_{1_9,-1_{10}} = |(\cos(\theta/2)\langle -1| - \sin(\theta/2)\langle 1|)|1\rangle|^2 = sin^2(\theta/2) \quad (3)$$

$$P_{-1_1 0,1_{11}} = \frac{1}{2}(cos(\theta) - sin(\theta))^2 \quad (4)$$

With similar probabilities for finding the -1 at 11AM. But the condition was that at 11 the the value was +1, so the probabilies for getting -1 at 11 are irrelevant. Thus the net probability for getting 1 at 10 would be

$$\begin{aligned} \hat{P}(1) &= P_{1_9 1_{10}} P_{1_1 0 1_1 1} / [P_{1_9 1_{10}} P_{1_1 0 1_1 1} + P_{1_9,-1_{10}} P_{-1_1 0,1_{11}}] && (5) \\ &= \frac{(sin(\theta) + 1)(cos(\theta + 1))}{2(1 + sin(\theta) cos(\theta))} && (6) \end{aligned}$$

because she must have gotten something at 10, ie the probability of getting something must be unity.

This is a perfectly well defined function of θ which has probability of 1 for $\theta = 0, \pi/2$ and 0 for $\theta = \pi, 3\pi/2$

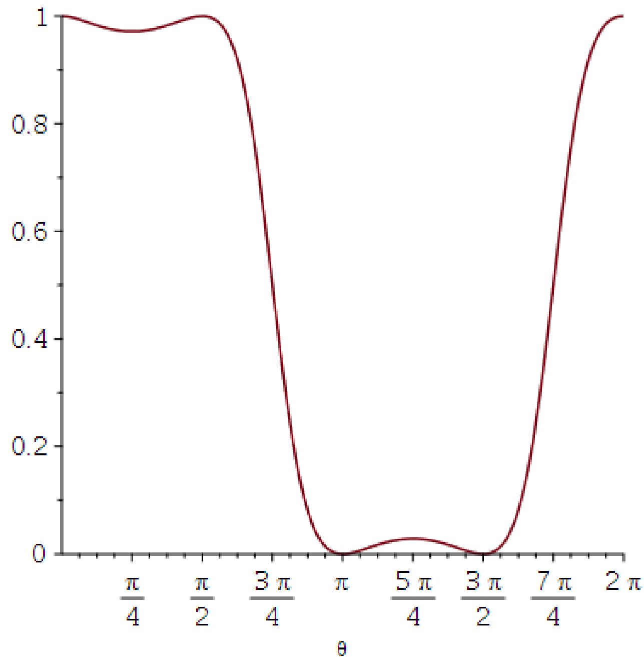

Figure 1: Probability for measuring σ_θ as 1 if $\sigma_3 \to 1$ ealier and $\sigma_1 \to 1$ later

This calculation is completely done using quantum mechanics. Ie, using conventional quantum mechanics one has found the probability for arbitrary values of θ but gives the probability of making a measurement at an intermediate time given conditions set at an earlier time and a later time.

Note that there is clearly no state or density matrix which would give a probability of unity for the two, non-commuting, operators σ_x and σ_y. Quantum Mechanics is fine. What is not fine is that the conditions one places on the system must be encoded in a single wave function. Instead we note that the answer could be obtained by using two wave functions, one setting the conditions at the earlier time, and one at the later time. Given a set $|\phi_n\rangle$ of orthogonal eigenstates belonging to some operator corresponding to some attribute of the system, the probability of getting the nth eigenvalue is

$$p(n) = |\langle\psi_{init}|\,|\phi_n\rangle\,\langle\phi_n|\,|\psi_{final}\rangle\,|^2 P(n) = \frac{p_n}{\sum_m p(m)} \tag{7}$$

Ie, one has to replace Born's rule by a more complicated rule for determining the probabilities. We note that this expression is symmetric under trading of ψ_{final} and ψ_{init}. Quantum Mechanics does not violate time sysmmetry.

This line of reasoning was carried out by Aharonov, Leibowitz and Bergman in the 1960's in the paper reference in the web page.

And this is all quantum mechanics. Heisenberg was wrong.