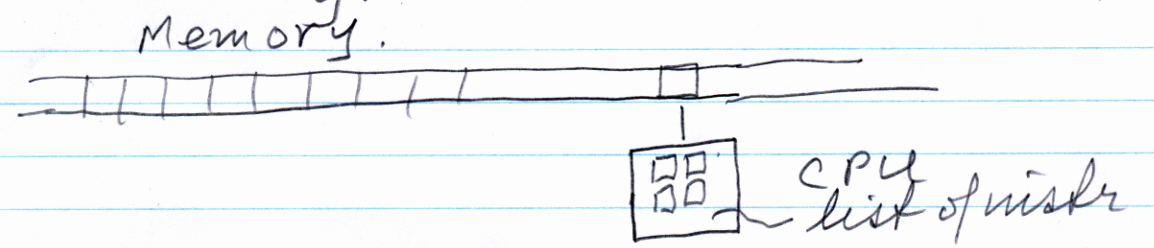
Varieties of Quantum Computing.

All Q. Computing based on Q bits (just as classical comp. based on bits). 2 level systems.

- excitations of atoms in 2 levels, electron or nuclear spin $1/2$ systems polarn of light, photons.

Turing machine.



Turing proved that any classical comp. could be carried out by this Turing machine and a few instructions. (advance tape, retard tape, alter memory and internal states depending on values of memory.

Q computer.

Memory - can be in a superposition.

A few instructions.

- Unitary transformations.

(a few instructions can replicate any Unitary matrix. operating on one or two Q bits at a time.)

All Unitary matrices are invertable $U|\psi\rangle = |\phi\rangle$

$|\psi\rangle = U^{\dagger}|\phi\rangle$

Erasure $|0\rangle \to |0\rangle$ $|1\rangle \to |0\rangle$?

Ancilla (extra bit) $|0\rangle$

$|0\rangle|0\rangle \to |0\rangle|0\rangle$
$|1\rangle|0\rangle \to |0\rangle|1\rangle$

then ancilla put away + never used again unless want to invert erasure

(Bit swap. $|0\rangle|0\rangle \to |0\rangle|0\rangle$
$|0\rangle|1\rangle \to |1\rangle|0\rangle$
Swap† = Swap. $|1\rangle|0\rangle \to |0\rangle|1\rangle$
$|1\rangle|1\rangle \to |1\rangle|1\rangle$

control not.

Not : $|0\rangle \to |1\rangle \quad |1\rangle \to |0\rangle$

control not : If first bit is 0 do nothing to second bit. If first bit is 1, do "not" to second

$$\text{CNot}: \begin{array}{l} |0\rangle|0\rangle \to |0\rangle|0\rangle \\ |0\rangle|1\rangle \to |0\rangle|1\rangle \\ |1\rangle|0\rangle \to |1\rangle|1\rangle \\ |1\rangle|1\rangle \to |1\rangle|0\rangle \end{array}$$

$$\text{Cnot}(\alpha|0\rangle|0\rangle + \beta|0\rangle|1\rangle + \gamma|1\rangle|0\rangle + \delta|1\rangle|1\rangle)$$

$$= \alpha|0\rangle|0\rangle + \beta|0\rangle|1\rangle + \gamma|1\rangle|1\rangle + \delta|1\rangle|0\rangle$$

$$\text{Xor} \quad |a\rangle|b\rangle = |a\rangle|\text{xor}(a,b)\rangle$$

(xor is a or b but not both

σ matrices $\quad \mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \mathbb{1}$$

$$e^{i\theta\sigma_a} = \sum_0^\infty \frac{(i\theta\sigma_a)^n}{n!} = \sum_n \frac{(i\theta)^{2n}}{2n!} + \sum_n \frac{(i\theta)^{2n+1}}{(2n+1)!}\sigma_a$$

$$= \mathbb{1}\cos\theta + i\,\sigma_a \sin\theta$$

Unitary

$$(\cos\theta - i \sin\theta\, \sigma_x)(\cos\theta + i \sin\theta\, \sigma_x)$$

$$= \cos^2\theta\, \mathbb{1} + \sin^2\theta\, \mathbb{1} = \mathbb{1}$$

$\sigma_x = \text{Not.} \quad \Rightarrow \quad e^{i \pi/2\, \sigma_x}$

Square root of not : $e^{i\pi/4\, \sigma_x}$

$$= \left(\frac{1}{\sqrt{2}}\mathbb{1} + \frac{i\,\sigma_x}{\sqrt{2}}\right)$$

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}} |0\rangle + i|1\rangle$$

$$|1\rangle \rightarrow \frac{1}{\sqrt{2}} |0\rangle - i|1\rangle$$

(sometimes it is also defined as $(e^{i\pi/4}\, e^{i\pi/4\,\sigma_x})$ but this is really $\sqrt{i}$.

Controled gates.

$$|0\ a\rangle = |0\ a\rangle$$

$$|1\ a\rangle = |1\rangle U|a\rangle$$

single bit unitary.

Square root of swap.

$|00\rangle \to |00\rangle \quad |11\rangle \to |11\rangle$

$|10\rangle \to |01\rangle \quad |01\rangle \to |10\rangle$

Square root of swap.

$|00\rangle = |00\rangle \quad |11\rangle \to |11\rangle$

$|10\rangle \to \frac{1}{2}\left((1+i)\, |10\rangle + (1-i)\, |01\rangle \right)$

$|01\rangle \to \frac{1}{2}\left((1-i)\, |10\rangle + (1+i)\, |01\rangle \right)$

Any one bit rotation plus sq rt of swap is universal. I.e any unitary can be created from these.

$\sqrt{\text{swap}}$ is entangling but not completely.

Hadamard gate. $\frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$

$$= (-i)\, e^{i\pi/2 \left(\frac{\sigma_x + \sigma_z}{\sqrt{2}}\right)}$$

Phase shift gate.

$$\text{phase} \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix} \overset{\text{Not}}{\overset{\downarrow}{\begin{pmatrix} 0 & 1 \\ +1 & 0 \end{pmatrix}}} \overset{\text{Phase}}{\overset{\downarrow}{\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}}} \overset{\text{Not}}{\overset{\downarrow}{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}}$$

$$= \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix} \text{ - overall phase.}$$

Adiabatic Quantum computing.
(Farhi et al 2000)

Adiabatic Thm.

$$i\,\underline{\partial_t} |\psi\rangle = H|\psi\rangle$$

$H(t)$ $\qquad H(t)|\psi_n, t\rangle = E_n(t)|\psi_n(t)\rangle$

$$|\Phi, t\rangle = \sum C_n(t)\,|\psi_n(t)\rangle$$

$$i\,\partial_t |\phi, t\rangle = H|\phi, t\rangle$$

$$\partial_t \sum_m C_n(t)\, e^{-i\int^t E_m\,dt'} |\psi_m, t\rangle = \sum_n C_n(t)\, E_n |\psi\rangle$$

$$\partial_t C_m(t) = -C_m\left(\langle\psi_m|\dot\psi_m\rangle\right) - \sum_{n\neq m} \frac{\langle\psi_m|\dot H|\psi_n\rangle}{E_n - E_m} e^{i\int(E_m - E_n)dt}$$

Last term is rapidly oscillating. I.e. $\dot H$ small, ($\langle\psi_m|\dot H|\psi_n\rangle \to 0$
& not oscillating on time scale of $1/E_n - E_m$ this goes to 0

$$\partial_t C_m(t) = e^{-\int\langle\psi_m|\dot\psi_m\rangle dt} \quad \text{Phase}$$

$$1 = \langle\psi_m|\psi_m\rangle \Rightarrow 0 = \langle\dot\psi_m|\psi_m\rangle + \langle\psi_m|\dot\psi_m\rangle = \langle\psi_m|\dot\psi_m\rangle^* + \langle\psi_m|\dot\psi_m\rangle$$

Pure phase.

As long as $e^{-i(E_n - E_m)}$

$(E_n - E_m)T >> 1$ during time

$E_n - E_m$ small. add n terms

cancel. $\frac{d}{dt}(E_n - E_m) << (E_n - E_m)^2$

If original is ground state.

Smallest value of $E_n - E_0$

determines how fast we can go & remain

in ground state.

Can easily create the ground

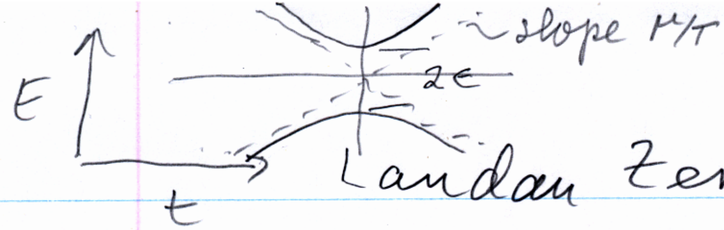
state. $|0\rangle|0\rangle \ldots |0\rangle$

$H = E\sum_i (i\sigma_z)$

Find Hamiltonian for which

ground state is solution of equations

you want



Landau Zenner.

$$H = \frac{\mu t}{T}\sigma_z + \epsilon\sigma_x = \overbrace{\sqrt{\left(\frac{\mu t}{T}\right)^2 + \epsilon^2}}^{E(t)}\left(\sin\theta\,\sigma_z + \cos\theta\,\sigma_x\right)$$

$$\tan\theta = \frac{t}{\epsilon T}$$

$$|-E\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}|1\rangle$$

$$|+E\rangle = \cos\theta|1\rangle - \sin\theta|\downarrow\rangle$$

$$|\psi\rangle = c_0|-E\rangle e^{i\int E\,dt} + c_1|E\rangle e^{-i\int E\,dt}$$

$$-i\partial_t|\psi\rangle = H|\psi\rangle$$

$$\partial_t c_1 = c_0\langle E|\partial_t|-E\rangle e^{2i\int E\,dt}$$

$$= \frac{d\theta}{dt} e^{2i\int E\,dt} c_0$$

To lowest order $c_0 = 1$

Landau, Zener showed that

$$|c_1^2| \approx e^{-2\pi\epsilon^2 T/\mu}$$

Exponential in T. (not power law.

If we go through the transition much slower than 1/the energy splitting prob of going out of the ground state exponentially suppressed.

(if we go through more rapidly, prob of excitation $\rightarrow 1$.)

$$H = H_0 (1 - t/T) + \frac{t}{T}(H_1)$$

at $t=0$, ground state is H_0,

at $t=T$, gnd state is H_1

If H_1 ground state is soln to your problem, then you have soln to high prob.

However. If somewhere between $t=0$, $t=1$, $\frac{t}{T}H_0 + H_1(1-t/T)$

has very small energy gap between ground + excited state, prob of not ending in ground state becomes ~ 1 unless T very large.

Grover problem.

$f(|n\rangle) = 1$ for $|\overbrace{10011101\cdots}^{n}\rangle$

except $f(|q\rangle) = 0$.

$H_1 |n\rangle |0\rangle = |n\rangle \; f(n)$

$H_1 |n\rangle |1\rangle = |n\rangle | g(q)\rangle \, (g(q) + E)$

(massively degenerate 1st level)

$H_0 = (1 + \sigma_z)/2 \implies$ ground state $|0\ldots\rangle$

$$H = \left(1 - \frac{t}{T}\right)\left(1 + \sigma_z/2\right) + \frac{t}{T} H_1$$

Show that the gap goes down to $e^{-\sqrt{N}/2}$

Time $> e^{N/2}$

Measurement Based Q. Comp.

(Raussendorf, Briegel) (2003)

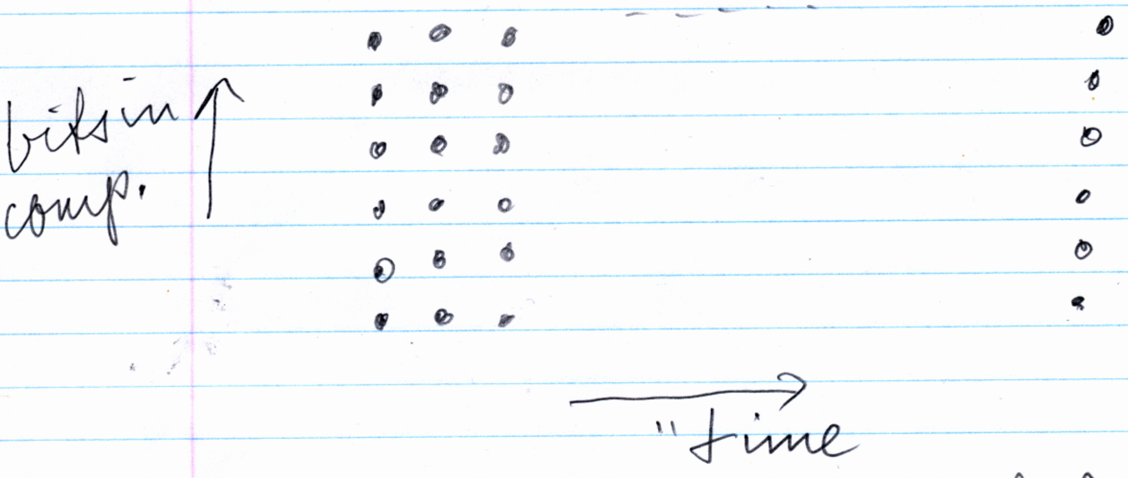
– Massive set of ~~s~~ 2 level systems.

(on direction is "time", other is operations).

Measure spins in various directions

Measure other spins depending on outcomes of first meas.

At end the meas give you the answer.



Can make many meas "out of time".