

Assignment 5
Physics of Music - 2016
Physics 341

1. Two loudspeakers are run so that the cones vibrate with the same velocity and the same frequency of 440. One of the cones is 5cm across, and the other is 10cm. How much difference in loudness (ie intensity of sound) is there between the two speakers? Express in both dB and relative intensities. (Remember both efficiency and size).

There are two effects here. The area of the 5cm speaker is only 1/4 that of the 10cm speaker. Thus the total sound produced at the speaker would be 4 times as big for the 10cm speaker (that is 6dB more) as for the small. Also the knee frequency of the 10 cm speaker would be 1/2 that for the 5. Thus any frequency below the knee frequency (which is 1700Hz for the 10cm speaker) would be an extra octave below the knee frequency for the 5 cm speaker, and thus an extra 6dB less efficient. Since 1/4 is the same as -6dB, the 5 cm speaker would produce 12dB less sound than the 10cm speaker.

2. In order to make her tuning fork more audible, Alice glues a piece of paper to one of the tines of her tuning fork. Why would this help? Would it be better or worse for her to glue a piece of paper to both tines?

The piece of paper does two things. One is that that tine's sound is no longer completely cancelled by the other. It also now has a bigger area (louder) and a lower knee frequency and is a double sided radiator. Ie, this will make it a lot louder.

If you glued on a piece of paper to both (assuming that both pieces were the same size) the sound from those two would again cancel. This is another case where more things trying to radiate make the sound quieter not louder.

3. A vibrating string on its own makes very little sound. Why? What is the purpose of the body of a guitar?

It is far too small to push around any air. It is a two sided radiator, is tiny and thus has a very high knee frequency (and the sound goes down at the rate of 12dB per octave).

The body is to move the air and make the sound. It is large (larger area, and much lower knee frequency) and thus can move air around, and make sound. The string's vibration vibrates the top of the guitar a tiny amount, but that is enough to make sound, while the string's huge vibration makes very little sound. (If the guitar top moved the same amount as the string, the intensity of the sound from a guitar would be far higher than 120dB.)

4)a) How much sharper or flatter (give a ratio) is a just major third (5/4) to two Pythagorean whole tones?

b) Three major thirds (four semitones) could be said to be an octave (twelve semitones). How mistuned would that octave be if each of those major thirds were just major thirds?

A Pythagorean whole tone is obtained by going up two major Pythagorean fifths, and then down an octave. I.e., $\frac{3}{2} \frac{3}{2} \frac{1}{2} = \frac{9}{8}$. Two Pythagorean whole tones are $\frac{9}{8} \frac{9}{8} = \frac{81}{64}$. But one just major third is $\frac{5}{4} = \frac{80}{64}$. Thus two Pythagorean whole tones are $\frac{81}{80} = 1.0125$ sharper than one just major third. Since a semitone is 1.058, this is just under a quarter of a semitone sharper.

Three just major thirds is $\frac{5}{4} \frac{5}{4} \frac{5}{4} = \frac{125}{64} = 1.953$. A semitone below an octave has a ratio of 1.888. Thus three just major thirds are just under half a semitone flat of an octave. If one tried to make sure that all major thirds were just major thirds, one would make a mess of the octaves.

5) In just tuning, what is the size (ratio) of the "semitone" between a minor third and a major third? What is the size of the semitone between a major third and a perfect fourth? (This variable size of semitones was already recognized by the Pythagoreans, who wondered about it and argued for it)

The minor third in Just tuning is a frequency ratio of 6/5 while a major third is 5/4. The ratio of these is $(5/4)/(6/5) = 25/24 = 1.042$ (compared with 1.059 of equal temperament). The ratio from a major third (5/4) to a perfect fourth (4/3) is $(4/3)/(5/4) = 16/15 = 1.066$ which is significantly sharper (larger) than just temperament and a lot sharper than the Minor-major third semitone (which difference is almost half a equal tempered semitone).

This large disparity in semitones is part of what gives just tuning its wide variety of sounds.

6) Two notes, tuned an equal tempered perfect fifth apart are played together. How many beats per second would you get between the first three harmonics of the two notes that have the same frequency in Just tuning if the lower note was 220Hz?

In just tuning every third harmonic of the lower note has the same frequency as every second harmonic of the sound a perfect fifth higher. This is not the same in equal temperament. The fifth is 1.4983 higher in frequency and thus would have a frequency of $220 \times 1.4983 = 329.63$ instead of 330Hz. The third harmonic of the lower note is 660Hz. The second harmonic of the higher is 659.26 Hz, so the difference is .74 Hz, which would give a beat frequency of .74Hz (about 3 beats in 4 sec).

The next harmonics which should have the same frequency in just tuning is the 6th of the lower note and the 4th of the higher. This would be a frequency of 1320 for the lower note, and 1318.52 for the higher, which would thus beat at about 1.48Hz (1 1/2 beats per second). I.e., the higher harmonics beat faster. The higher harmonics tend to be quieter but the ear still hears this as a change in the tone colour of the note and a slight harshness to the chord.