Physics 200-04

Measurement

Change of knowledge

Let us say that we have our two level system, and someone hands us such a system in a state $|\psi\rangle$. We now determine by some manner its energy, represented by $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$. The probability that the answer will be E_1 is then given by $|\langle E_1||\psi\rangle|^2$. After we have determined the energy, what is the state of the system? That state is $|E_1\rangle$. Is there anything left of the original state ψ? The answer is no. Having determined the energy, whatever the state was originally has disappeared, and is no longer or any relevance to the future of the two level system.

This procedure is known in the literature as "the reduction of the wave packet". It is simply the change in the state of the system occassioned by the new knowledge, the energy E_1 of the system.

Note that new knowledge in this case does not add to the old. It replaces the old knowledge for the future.

Probabilities Let us say that we have two different systems, 1 and 2. In both of these systems the probability of energy E_1 is 1/2. Does this completely characterise the system? No.

system 1 is prepared in the following way. The person has two huge bags full of identical two level systems, one bag is full of systems with energy E_1 and the other of systems with energy E_2. He flips a coin and hands you the systems from one of the two bags depending on what the outcome of the coin flip was.

System 2 on the other hand is prepared by the person handing you a two level system prepared in the state $|\psi\rangle = \frac{1}{\sqrt{2}}(|E_1\rangle + |E_2\rangle)$. According to the above rules, this system will also have probability of 1/2 of being in the energy state E_1, and nothing about the energy can differentiate between these two systems.

However, if instead of measuring the attribute $H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix}$, with eigenvectors $|E_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|E_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ we determine the attribute represented by the matrix $S = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, whose eigenvalues are ± 1 and eigenvectors are $|S,1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{\sqrt{2}}(|E_1\rangle + |E_2\rangle)$ and $|S,-1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix} = \frac{1}{\sqrt{2}}(|E_1\rangle - |E_2\rangle)$

In the first case, the probability of measuring S to have value 1 is, if the coin toss said that type E_1 was chosen, $|\langle S,1||E_1\rangle|^2 = \frac{1}{2}$. Similarly, if type E_2 was chosen by the coin toss, then the probablitity that S would have value 1 is $|\langle S,1||E_2\rangle|^2 = \frac{1}{2}$. In each case the probability that S has value 1/2 is 1/2 and the total probability that S would have value 1 is 1/2.

However in case 2, the system is in the state $|\psi\rangle = |S,1\rangle$.Thus the probability that one would find S to have value 1 is $\langle S,1||\psi\rangle = \langle S,1||S,1\rangle = 1$. Ie, the probability is unity now that S has value 1.

Ie, two systems which are identical as far as the energy is concerned, are not identical as far as the property S is concerned. This combination of energy eigenstates with given probabilities into a state in which the probability of some other attribute, S, does not simply have the combination of probabilities of S have property 1 in each of the energy states, is called interferences. Ie,

$$ProbS = 1 \neq Prob(S = 1|E_1)Prob(E_1) + Prob(S = 1|E2)Prob(E_2) \quad (1)$$

which one would naively expect of probabilities in a classical situations. (The notation $Prob(S = 1|E_1)$ means the probability that S will have value 1 given that the energy has value E_1. Ie, here the vertical bar | means "given that") Quantum probabilities behave somewhat differently from classical ones.

This property of interference is one that we are familiar with in the case of waves. The intensity of the light coming from two slits for example is not simply the sum of the intensities coming from each slit. It will depend on whether one is at a position where crest meets trough, or crest meets crest. In one case the intensity could be zero, and the other it could be twice as big as simply the sum of intensities. Ie, the intesities of the wave act like the probabilities in the quantum system.

Using this analogy, the coefficients in the expansion of a state in terms of the eigenvectors of some attributs

$$|\psi\rangle = \langle E_1||\psi\rangle|E_1\rangle + \langle E_2||\psi\rangle|E_2\rangle \quad (2)$$

The in general complex coefficients $\langle E_1||\psi\rangle$ and $\langle E_2||\psi\rangle$ are called the amplitudes of the state $|\psi\rangle$ in terms of the Energy eigenstates.

It is this similarity between interference between waves, and interference in quantum mechanics that leads to the statement that quantum systems have wave-like properties. They have wavelike properties in that the two possibile values for an attribute for a system (the energy in the above example) can interfere so as to alter the probabilities for the values of some other attribute.