Physics 200-06
Final Exam
Dec 15, 2006 3:30 PM to 6:00 PM

This exam consists of three pages, three sections, and 8 questions. There is a Relativity section (questions 1 to 3) and a Quantum Mechanics section (Questions 4 to 7), and an "either" section, question 8.

You are to answer two (2) questions on Relativity and three (3) on the Quantum mechanics (i.e., five (5) in total). There is one question– number 8– which can be used in either section, ,i.e., which may be used either as one of the two questions from the Relativity section or one of the three questions you answer from the Quantum section, but not both.

All problems are worth the same number of marks.

If you answer more that 5 you must tell me which ones are to be marked, or I will simply take the first 5 that you answer (consistent with the relativity/quantum split).

Relativity

1.a) A particle of mass M at rest emits a gamma ray of energy E, leaving a particle of mass M'. What is M' as a function of E and M?

b)Now the resultant particle of mass M' in part a) at rest. What energy ϵ would a gamma ray particle need in order to be absorbed and create the original particle of mass of M again. Why is ϵ not the same as E?

2.) a) Pole and Barn revisited. A runner carrying a horizontal pole is running at a barn at 4/5 the velocity of light. When both are at rest, the pole has exactly the same length as the barn. According to the runner, the barn will be contracted. However, when the runner, located at the trailing edge of the pole, **sees** the front of his pole hit the far inside end of the barn, how far from the end of the barn is he located?

b) Bob leaves Alice behind on earth and travels off to the nearest star 4 light years away and returns. (A light year is the distance light travels in one year) On his return, Bob finds that the total trip according to his clocks is 10 hours less than Alice claims it was. How fast was he travelling? (Assume constant velocity for the trip there and back). (Note– Use 1 year=10^4 hours.)

3.) a)Discuss the history of the development of Special Relativity in some detail.

b) Light traveling through water at rest travels at a speed of $\frac{1}{n}$ the velocity of light where n is the index of refraction. Assume that the water is traveling at velocity v in the lab in the same direction as the light is traveling. What, to lowest order in v is the velocity of that light according to the lab frame.

Quantum Mechanics

4.)a) Using the Bohr Sommerfeld quantisation rules, find the energy levels for a particle in a one dimensional box of length L where the particle collides with the walls of the box elastically.

b) Briefly describe the development of quantum mechanics from 1900 to 1925 touching on some of the key ideas and incidents during that period.

5.)a) Consider the two attributes, A and B, represented by the matrices

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{1}$$

$$B = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \tag{2}$$

What are the eigenvalues and eigenvectors of these two matrices?

If attribute A is measured and found to have its largest eigenvalue, what are the probabilities for measuring the two possible values of B?

If B is now found to have its largest value, what is the probability for then measuring the largest value for A?

b) Consider the state

$$|\psi\rangle = \begin{pmatrix} 1 \\ i \end{pmatrix} \tag{3}$$

. Is this normalized? If not, normalize it. What are the expectation values of the three σ matrices in this state? What are the probabilities of measuring the value +1 for each of the attributes represented by the three sigma matrices in this state?

6.)a) If A and B are two attributes, and $|a\rangle$ is an eigenvector of A with eigenvalue a, what is the expectation value of the commutator of A and B, $\langle a|[A,B]|a\rangle$?

b) Assume that X is the matrix representing the attribute of position of a particle, and P is the attribute of momentum, which Heisenberg showed obeyed

$$[X,P] = i\hbar I \tag{4}$$

use the result from part a) to argue about the existence or not of the eigenvectors $|x\rangle$ of the matrix X.

c) Show that $[X,P^n] = i\hbar n P^{n-1}$. You can prove this via induction– eg, show that if $[X,P^n] = i\hbar n P^{n-1}$ for some specific n then it must also be true for $[X,P^{n+1}] = i\hbar(n+1)P^{n+1}$. (Use the knowledge you have of the commutator of $[A,BC]$ in terms of the commutators of $[A,B]$ and $[A,C]$.

7.) The energy attribute for a two level system is given by

$$H = \frac{\epsilon}{2}\sigma_3 \tag{5}$$

A particle starts out in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{6}$$

the eigenstate for the attribute σ_1. What will the state be as a function of time? What will the expectation of the attribute σ_1 be as a function of time?

Assume that particle is a neutrino, and the attribute σ_1 represents the whether the neutrino is an electron type neutrino (interacts only with electrons), or a muon neutrino (interacts only with muons), and the energy difference ϵ in the above is about 10^{-30} Joules. How fast would that neutrino have to travel in order that it changed once from electron-type to muon-type neutrino in traveling from the Sun to the earth (1.5×10^{11} m) (ignore relativity for this problem– is that a valid approximation?)

Question for either section

Note that if you answer this question, it replaces one of the choices in Relativity or Quantum Mechanics. It cannot be used in both sections however. The total number of questions you are to answer is 5.

8.) Make up your own question on either Quantum Mechanics or Relativity, and answer it. It must not be the same as any of the other questions already on this exam. You will be marked both on the quality and difficulty of the question and on your answer (I.e., a trivial question with simple answer will not get many marks, nor will an impossible question with no answer). The question must be on either Relativity or Quantum Mechanics and counts as one of the questions in that section. This is an opportunity to show me what you know that I might not happen to have asked a question about on this exam.

$$\begin{aligned}
c &= 3\times 10^{8}\ \mathrm{m/s} \qquad h = 6.6\times 10^{-34}\ \mathrm{Js} \qquad \hbar = 1.1\times 10^{-34}\ \mathrm{Js} \\
\hbar &= 6.6\times 10^{-25}\ \mathrm{GeV-sec} \\
\sigma_1 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\end{aligned}$$