Physics 200-04
Assignment 4

1) [Based on French 6-8]. A propulsion system has been proposed where a strong llaser is shone at a totally reflecting "sail" in space. The sail is assumed to be perfectly reflecting in its own rest frame. Ie, the energy of the photon reflected equals the incident energy in this frame (assuming that the rest mass energy of the sail is much greater than the energy of the photon) Ie, you can assume that in the frame the sail, the photon has the same energy after reflection as when it was incident.]

i) First, assume that the sail is much heavier than the particle. Show that if the sail is travelling with velocity v, the energy transfered to the sail by a single photon of incident energy ϵ (travelling in the same direction as the sail) is $2\epsilon\frac{v}{1+v}$.(This problem uses coordinates such the c=1) (Hint– transform the photon to the frame moving with the sail, assume specular reflection and then transform the reflected photon back to the original frame.)

ii) Consider the photons emitted from the source at n per second. How many photons per unit length are there travelling from the source to the sail? What is the total number between the source and the sail when the sail is a distance x from the source. How many photons per second hit the sail in the frame of the source? What is the energy transfer per unit time to the sail?

2) Consider a mass, mass M, which emits a particle of mass m and leaves the large particle with mass M'. Show that for given M and M', the velocity of the mass M' is largest as m goes to zero. Ie, converting given mass to photons is the most efficient way of using that mass as a fuel for accelerating the large mass

3) A spaceship converts a certain fraction x of its mass into light per second (as measured by the spaceship itself) and shines the light out the back of the spaceship to accelerate it.

i) Show that the acceleration of the spaceship is constant in magnitude (ie $\bar{a}\cdot\bar{a} = |a|^2$ is constant, where $\bar{a} = \frac{d\bar{u}}{d\tau}$ is the acceleration four vector.

ii) How much of the original mass must be used up in order that the final mass of the spaceship is travelling at 0.99c

[Note: For neither of these parts do you need to solve differential equations.]

4) Scattering: A particle of mass m and velocity v collides with a larger mass M at rest. After the collision, the two masses are still the same, and the small mass moves at an angle θ with respect to its original motion. What is the velocity of the large mass after the collision.

5)Are the following possible, and if not, why not.

i) A gamma ray (which moves at the speed of light)of energy 1.5MeV decays into an electron and positron, each having the same mass, .5MeV. (Note the convention in particle physics is to measure both energies and masses in eV (electron Volts). One electron volt is $1.6\ 10^{-19} J$.

ii) Two gamma rays each of energy 1GeV collide to produce and electron and a positron.

iii) A particle of mass M at rest collides with a gamma ray. After the collision the gamma ray is absorbed and the resultant particle still has mass M, and some non-zero velocity.

iv) proton of Mass .9383 GeV decays into a neutron of mass .9395 GeV plus a positron of mass .5MeV and a neutrino of mass 0.

v)One of the above neutrons decays into a proton, and electron, and a neutrino with masses as above.