Physics 200-04
Assignment 1

1. Aberration: Fill in the details of Bradley's argument. Ie, assume that gamma-Draconis is directly overhead London (latitude 52) and that the angle of the earth's axis to the plane of the orbit is 23 degrees. Thus, the angle between the direction to the star and the earth's orbit is 75 degrees. Given the earths velocity in its orbit is 30km/s and the velocity of light is 300,000km/s. What would the aberration angle (the change in angle to the star due to the motion of the earth) be? Bradley believed that he could measure changes in angle of 1 second of arc. Is the aberration measurable?

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[5] The first item is that Bradley could not measure east-west angles. He had no accurate clocks, so had no idea at what time the star was supposed to be straight overhead. (The earth rotates at $360 \times 60 \times 60 = 1.3 \cdot 10^6$ sec of arc/ 10^5 sec=12 sec of arc/sec, and with clocks which were maybe accurate to only a few seconds per day, the accuracy east west was poor). Thus he could only measure north-south angles with any accuracy. North is in the dirction in which the earth's axis tilts at 23 deg to the plane of the orbit. London is at 52 degrees north lattitude, and thus the total angle of straight up in London to the plane of the earth's orbit is 75 degrees. When the earth has its max velocity in the northerly direction, the 30 km/sec velocity of the earth along its orbit vectorialy adds to the velocity of light. By the cosine law , the square of the velocity along the unlabeled sice is

$$c'^2 = (3 \cdot 10^5)^2 + (30^2) + 2cos(75)(30(3 \cdot 10^5) \approx (3 \cdot 10^5)^2 \tag{1}$$

Ie the speed is almost exactly the same to one part in 10^4. By the sin law,

$$\frac{sin(\delta\theta)}{30} = \frac{sin(75)}{3 \cdot 10^5} \tag{2}$$

or, since $\delta\theta$ is very small (in radians) we have

$$\delta\theta \approx 10^{-4} sin(75) = 9.7 \cdot 10^{-5} = 19.9 \; seconds \tag{3}$$

which is certainly measureable.

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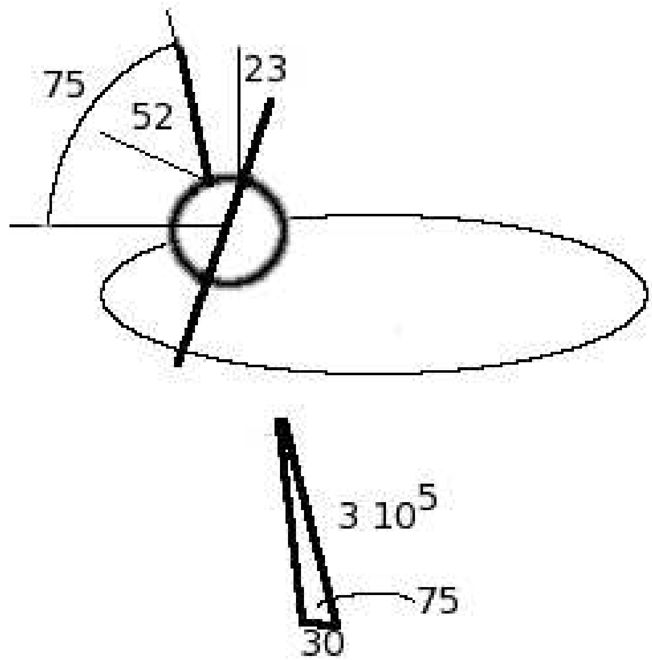
2. Text book Problem 2-2: The following is a sample of the data from the Fizeau experiment as repeated in 1886 by Michelson and Morley:

Wavelength of light 5700 Angstrom
Length of each tube 6.15m
Velocity of water flow 7.65 m/sec
Mean fringe shift upon reversal of flow 1.72 ± 0.01
Compare the value of the "drag coefficient" implied by these data with the value $1 - \frac{1}{n^2}$ for water (n=1.33)



[5]– Note that it is very easy to forget that the question asks for the fringe shift on the reversal of the flow of the water, not between the flow and no flow.

Flowing with the water, the light travels with velocity $c/n + v(1 - 1/n^2)$, and the total length is $2L$, so the time is $2nL/(c + nv(1 - 1/n^2))$. Flowing against the stream, the light velocity is $c/n + v(1 - 1/n^2)$ and the time is $2nL/(c - nv(1 - 1/n^2))$. Thus the difference in the times is

$$\Delta T = \frac{4n^2Lv(1-1/n^2)}{c^2 - (nv(1-1/n^2))^2} \quad (4)$$

$$\approx \frac{4n^2Lv(1-1/n^2)}{c^2} \quad (5)$$

plugging in the values of $n = 1.33$, $v = 7.65m/s$, $c = 3 \cdot 10^8 m/s$, $L = 6.15m$, we get

$$\Delta T = 1.6 \cdot 10^{-15} \quad (6)$$

This corresponds to a wavelength difference of

$$\frac{c\Delta T}{\lambda} = .85 wavelengths. \quad (7)$$

But this wavelength difference is just the difference between the water flowing in the one direction, not the difference on reversal of the flow. When you reverse the flow, the role of the two directions that the light travels is reversed, and you get a difference in wavelength difference in the opposite direction when the flow is reversed. Thus the wavelength difference on reversal of the flow, and the fringe shift is 1.7 wavelengths(double the above amount) which agrees with their measured value.

3. Maxwell suggested measuring the velocity of the sun through the aether by timing the exact time of the eclipse of one of the moons of Jupiter when Jupiter was in various orientations with respect to the fixed stars and when the earth was nearest to and furthest away from Jupiter. From the observations, the extra time it takes light to cross the orbit of the earth would depend on the speed of light. (This was Roemer's method for measuring the velocity of light) Now, if the sun traveled through the aether with velocity v, then the velocity of light as measured by the Roemmer experiment would be c+v if Jupiter were in front of the sun (with respect to its traveling through the aether) and c-v if behind. Estimate the magnitude of the velocity v that could be measured in this way, now and in 1850. (You will need to know what the best accuracy of clocks was then and now. Use google to try to find the history of the accuracy of the clock for example. You will also have to estimate how accurately you could measure the time of the eclipse of the moon by the planet Jupiter.)

[5]– Roemer's method is to measure the velocity of light by measuring the time it takes to cross the orbit of the earth. This is done by measuring the time between the eclipses of the moon of jupiter on one side of the orbit, and then using that to predict the time at which the eclipse should occur for any orbit of the moon. By measuring the difference between the time of the eclipse from this prediction, Roemer could calculate the velocity of light.

In the Maxwell suggestion, he assumed that the sun travelled through the aether, and thus, if Jupiter was ahead of the sun, the light would get to the earth faster, by a relative velocity of c+v, where v is the velocity of the sun. When Jupiter is behind the sun as far as the direction of motion of the sun, the light must catch up to the earth, and its relative velocity is c-v. The time across the orbit of the earth is approx 16 min= 1000sec if the velocity v is zero. The difference will be

$$\frac{2R}{c-v} - \frac{2R}{c+v} = \frac{4Rv}{c^2-v^2} \approx 2 \cdot 10^3 sec\frac{v}{c}. \tag{8}$$

using the radius of the earth's orbit of $1.5 \cdot 10^8 km$ and $c = 3 \cdot 10^5 km/s$.

Now, how accurately could one measure the time? To measure the velocity of light at any time takes just over half a year for the earth to move from nearest point to jupiter to the furthest. Thus the question is how accurately could one time the time of the eclipse, and how accurately could one time the time between the eclipse when teh earth was on one side to the other.

This depends on two factors– how accurate clocks were and how accurately one could actually time exactly when the eclipse occured. In the 1850s, clocks had an accuracy of about 1/100 of a second per day. This would correspond to about 1 sec of acuracy per half year. Thus an estimate of the accuracy with which one could time the orbit was about 1 second. An estimate of the accuracy with which one could observe the eclipse is more difficult. The best is probably how long it would take the moon to move its own radius (ie, how long it would take to actually move behind the rim of the planet). The fasterst moon is Europa, with a period of 1.8 days, a diameter of 3600km, and an orbit of 421,600km. This gives a time of about 200 sec for the moon to disappear behind jupiter. It is highly unlikely that one could at that time measure the time of eclipse to better accuracy than this. (The moon is such a small dim object disappearing behind a bright planet, making it very hard to determine exactly how the light disappeared behind the planet). Ie, the accuracy of determining exactly when the eclipse occured would have been far worse than the accuracy with which it could be timed. Taking 200 or about 3.5 min as the accuracy, We would get a measureable v/c of about .2. Ie the sun would have to be travelling at very close to the velocity of light.

Even at the present, with space based telescopes, the accuracy with which one could estimate the time at which the moon disappeared behind jupiter to not much better than say 1/10 of the diameter of planet, or about 20 sec. This is still only a speed of the sun of .02 c, which is absurd.

If one could put an active satellite orbiting Jupiter, the accuracy of measuring the eclipse (and teh actual orbital parameters and their depence on time as seen from the earth would be much much higher. The estimated orbit of the sun around the galaxy is about 300 m/sec and the estimated velocity of the earth against the frame of the cosmic microwave background is about 600km/sec or $v/c \approx .002$. This would have shown up in the measurement of the time from various satellites sent to orbit Jupiter, etc. It did not. No measurement of the Maxwell kind has ever detected a motion by seeing a difference in the velocity of light.

Note that as often occurs, some of the students came up with an additional note, and that is that the time delay can actually be increased by always looking at the earth at its furthest point from Jupiter over one whole orbit of Jupiter, rather than just one orbit of the earth. Since $R_J + R_E \approx 6R_E$, the baseline over which one is measuring the velocity of light is actually 3 times further than just using the Earth's orbit to measure the velocity of light along. This would increase the accuracy by a factor of 3. HOwever, the minimal velocity measurable would still be absurdly large, given the difficulty of timing exactly when the eclipse occured.

. 4. Galilean Relativity and Maxwell's Force law:

Consider the following force law for a charged particle with charge e moving through a magnetic field $\vec{B}$ and electric field $\vec{E}$.

$$\vec{F} = e\vec{v} \times \vec{B} + e\vec{E} \tag{9}$$

where $\vec{v}$ is the velocity of the particle. We know that the left hand side of Newton's equations is invariant under Galilean transformation with constant velocity $\vec{v}_0$. Under such a transformation, the position of the particle changes by $\tilde{\vec{x}} = \vec{x} + \vec{v}_o t$. How does its velocity change? Is the force invariant under this transformation? Can one change one's definition of $\vec{E}$ and $\vec{B}$ so as to make the force be the same after the transformation?

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[5] – Crucial items are deriving what tthe force would be in the moving frame and noticing that if the E field were redefined you would get the same force law as in the other frame.

Under an ordinary galilean transformation (ie constant velocity $\vec{v}_0$, the velocity of an object changes from $\vec{v}$to $\tilde{\vec{v}} + \vec{v}_0$. The left hand side of Newton's force equation $ma = F$ we know does not change under such a transformation. However, we find that the force goes from

$$\vec{F} = e\vec{v} \times \vec{B} + e\vec{E} \tag{10}$$

to

$$\vec{F} = e(\tilde{\vec{v}} + \vec{v}_0) \times \vec{B} + e\vec{E} \tag{11}$$

If $\vec{E}$ and $\vec{B}$ did not change during the transformation, we would find that this Lorentz force law for a charge would not obey invariance under the constant velocity transformation. The force would be different in the different frames.

However, we can rewrite this as

$$\vec{F} = e(\tilde{\vec{v}} \times \vec{B}) + e(\vec{v}_0 \times \vec{B} + \vec{E}) \tag{12}$$

This suggests that we can maintain the invariance of the force law, if in the new frame we assume that $\vec{B}$ remains the same, but that the new electric field

$$\tilde{\vec{E}} = \vec{E} + \vec{v}_0 \times \vec{B} \tag{13}$$

Ie, the electric field and the magnetic field are objects that change if you go into a different moving frame.

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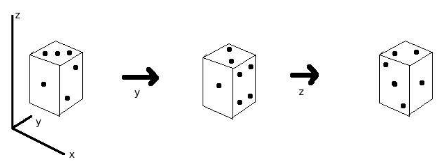
5. Show that a rotation by 90 degrees of a body about the y axis, followed by a rotation of 90 degrees about the z axis is not the same as first rotating by 90 degrees about the z axis and then by 90 degrees about the y axis. The physical operations of rotations do not commute. (Two operations commute if it does not matter which order they are done in).

[4]– this problem can be answered in many ways. Matrix multiplication is one, but diagrams is fine.

This is most easily seen from diagrams. In the following is a dice. (recall that the sum of the opposite sides of a dice always add up to 7. Thus the three spots are opposite the four, the two opposite the five and the one opposite the six). Note that the final orientation of the dice is not the same in the two cases.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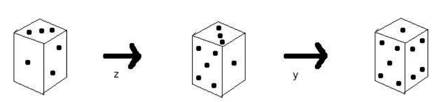
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