Physics 200-04

Supplimentary Problem 2

Old quantum theory.

Given a particle in a well, such that the potential enegy is 0 for $-L < x < L$ and infinity outside this range. What are the energy levels?

Solution:

The old quantum condition is that $nh = \int p\dot{x}dt = \int pdx$. The energy is

$$E = \frac{1}{2}mv^2 + V(x) \tag{1}$$

Thus for all energies less than infinity, the particle must be in the range $-L < x < L$. In this range we have

$$\begin{aligned} \frac{1}{2}mv^2 = \frac{1}{2m}p^2 = E \\ p = \sqrt{2mE} \end{aligned} \tag{2}$$

Thus the quantization condition becomes

$$nh = \int_{-L}^{L} \sqrt{2mE}dx + \int_{L}^{-L} -\sqrt{2mE}dx = 4L\sqrt{2mE} \tag{3}$$

or

$$E = \frac{n^2h^2}{32mL^2} \tag{4}$$

In this case the answer is exact. The energy levels are exactly these in the true quantum theory as well.

What if the well is not infinite? For total enegies less than the top of the potential enegy the Bohr Sommerfeld rules would give exactly the same value for the energies as above. However, in the exact QM, the energies drift from the above, so that for energies very near the top of the potential, the energies have decreased to lie almost exactly between the two levels given by the above expression. Ie, sometimes the Bohr Sommerfeld gets it right, and sometimes not. What is astonising is that for the hydrogen atoms the rules got it right. Otherwise quantum mechanics might never have been discovered.