Physics 200-05
Practice 2

1)From the definitions

$$cosh(\theta) = \frac{e^{\theta} + e^{-\theta}}{2} \quad (1)$$

$$sinh(\theta) = \frac{e^{\theta} - e^{-\theta}}{2} \quad (2)$$

show that $cosh(\theta)^2 - sinh(\theta)^2 = 1$ and find the derivatives of both $cosh(\theta)$ and $sinh(\theta)$.

If $\tanh(\theta) = \frac{sinh(\theta)}{cosh(\theta)} = \frac{v}{c}$, find $cosh(\theta)$ and $\sinh(\theta)$.

The first part is just a bit of algebra. The second (derivatives) use that

$$\frac{de^{\theta}}{d\theta} = e^{\theta} \quad (3)$$

$$\frac{de^{-\theta}}{d\theta} = -e^{\theta} \quad (4)$$

to give

$$\frac{d\cosh(\theta)}{d\theta} = sinh(\theta) \quad (5)$$

$$\frac{d\sinh(\theta)}{d\theta} = cosh(\theta) \quad (6)$$

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2)Find the expression for the Lorentz transformation to first order in $\frac{v}{c}$.

$$\tilde{x} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(x - vt) \approx x - vt \quad (7)$$

$$\tilde{t} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(t - \frac{v}{c^2}x \approx t - \frac{v}{c^2}x \quad (8)$$

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3. What would the Lorentz transformation with velocity v in a direction at 30 degrees from the x axis in the x-y plane? In the perpendicular direction to

the motion there is no change. Thus

$$\tilde{z} = z \quad (9)$$

$$cos(30)\tilde{y} - sin(30)\tilde{x} = cos(30)y - sin(30)x \quad (10)$$

In the direction along the direction of motion (which would correspond to the direction $cos(30)x + sin(30)y$) we have te uaual transformation

$$cos(30)\tilde{x} + sin(30)\tilde{y} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(cos(30)x + sin(30)y - vt) \tag{11}$$

$$\tilde{t} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(t - \frac{v}{c^2}(cos(30)x + sin(30)y)) \tag{12}$$

In general, we can write the transformation as

$$\vec{v} \cdot \vec{\tilde{x}} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(\vec{v} \cdot \vec{x} - v^2 t) \tag{13}$$

$$\tilde{t} = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}(t - \frac{\vec{v} \cdot \vec{x}}{c^2}) \tag{14}$$

$$\vec{v} \times \vec{\tilde{x}} = \vec{v} \times \vec{x} \tag{15}$$

Ie, the components which are perpendicular to $\vec{v}$ are unchanged, while those parallel are transformed.

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4. What would the expression for distance be in three dimensions if the z directions are measured in feet, and x,y in meters? What would the expression for rotation by angle θ about the y axis be.

If we write g as the transformation from feet to meters– ie .305 m/f, then we would have

$$(\text{distanceinmeters})^2 = x^2 + y^2 + g^2 z^2 \tag{16}$$

Ie, we get a conversion factor in the distance formula which converts the z coordinates to the same units as the x and y. We would get that same conversion factor occuring in rotations. For example rotating around the y axis, by angle θ we would get

$$\tilde{x} = \cos(\theta)x + \sin(\theta)gz \tag{17}$$

$$\tilde{z} = \cos(\theta)z - \frac{1}{g}\sin(\theta)x \tag{18}$$

Ie, we would get extra factors of g or $\frac{1}{g}$ occuring in all of the equations. Note the similarity to the factors of c which occur in the equations of Lorentz transformations. If we measured all times in the same units as space (eg meters), then all of the factors of c would vanish from the Lorentz formulas and they would look much simpler.