Physics 200-05
Midterm Exam
Oct 24 2005
Time: 1:00PM to 1:50PM

This exam consists of five (5) questions. All problems are worth the same number of marks. [4] marks each.

Note: You can use

$$c = 3 \cdot 10^8 m/sec$$

1. Given that the Matricees A and B are given by

$$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{1}$$

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2}$$

Calculate the matrix

$$(AB - BA)^T \tag{3}$$

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[2]

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{4}$$

$$BA = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{5}$$

$$AB - BA = 2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{6}$$

$$(AB - BA)^T = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \tag{7}$$

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b) Consider the displacement 4-vector

$$\bar{p} = \begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

Argue that if this represents a null (lightlike) displacement, then a Lorentz transformation takes this null displacement into another null displacement. Show that if the Lorentz transformation is in the x direction and the null displacement is in the x direction (ie, $\Delta y = \Delta z = 0$) that this null displacement vector is taken into a multiple of itself.

Since a Lorentz transformation leaves the 4-distance the same after a transformation, if the 4-distance is 0 before the transformation it is 0 afterwards as well. [1]

For a null displacement in the x direction we have

$$\Delta x^2 - \Delta t^2 = 0 \tag{8}$$

of $\Delta x = \pm\Delta t$, Thus the displacement vector is

$$\begin{pmatrix} \Delta t \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} = \Delta t \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

Under a Lorentz transformation in the x direction, Δy and Δz do not change so they remain 0. Δt and Δx do change but since the displacement is still null, we still have $\Delta\tilde{x} = \pm\Delta\tilde{t}$, with the sign of the $\pm$ the same as before. Thus the new transformed displacement is

$$\Delta\tilde{t} \begin{pmatrix} 1 \\ \pm 1 \\ 0 \\ 0 \end{pmatrix} = \frac{\Delta\tilde{t}}{\Delta t}\bar{p} \tag{10}$$

[1]

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2) Far in the future, a runner in the 120 kilometer dash disputes his record breaking time with the official time-keeper. He says that according to his own watch, which he carried with him during the race, his time for the race was only 80% of the official time-keeper's time. What was the official time for his running of the race? (Assume that all clocks were accurate clocks and the runner is assumed to have run the race at a constant speed throughout.)(Yes, the answer is supposed to be absurd).

Since the runner must be running so fast that his time is dilated to only .8 of its original[1], we must have that

$$\sqrt{1-v^2} \quad = \quad .8 \qquad [1] \tag{11}$$

or

$$v^2 = .36 \rightarrow v = .6 \qquad [1] \tag{12}$$

Thus he must have been running at $.6c$ Since the track was 120km long, the official time must have been

$$\frac{120km}{.6 \cdot 3\ 10^5 km/s} = .666 \cdot 10^{-3} sec \tag{13}$$

[1]

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3) Consider two observers on the surface of the earth, at opposite sides of the earth at the equator. They both see a flash of light on the (rising/setting) moon. What is the difference in time that they will ascribe to when the flash occured on the moon? The earth's diameter is 6000 km, it rotates once every 24 hours, and the distance from the earth to the moon is 400,000 km.

The two observers on the opposite sides of the earth both see the moon. The only time is when it is moonrise for one and moonset for the other, since otherwise teh earth would get in the way of both seeing it.

It is of course the earth's radius that is 6000 km., not its diameter.

In the solutions below, I use this, but if you use my wrong assumption about the earth above, then all velocities are half and the time synchronisation difference is also half of what it is.

The velocity of the observers is

$$v = 2\pi \frac{6000km}{24 \cdot 60 \cdot 60 \; sec} \approx .42km/s \tag{14}$$

Thus, $\frac{v}{c \approx .14 \cdot 10^{-5}}$. [1]

The distance to the moon is 400000km, and the time synchronization from teh static earth for each is

$$\tilde{t} = \frac{1}{\sqrt{1 - (.14 \cdot 10^{-5})^2}} (t \pm \frac{v}{c^2} x) \tag{15}$$

[2] where the sign depends on whether the observer is moving toward or away from the moon. The prefactor is irrelevant (it makes a change of only 1 part in 10^{11} so the time difference is

$$2\frac{v}{c^2} x \approx \frac{.28 \cdot 10^-5}{3 \cdot 10^5 km} 400000km \approx .38 \cdot 10^-5 sec \tag{16}$$

[1]

4) Peter Spacerider has heard about Relativity and heard that from the point of view of a rapidly travelling observer, his own spaceship is really short. He passes a spaceship identical to his own travelling in the opposite directioni at almost the velocity of light. Just as the nose of his spaceship is at the tail of the other spaceship he presses the button in the nose to fire the laser canon in the tail of his own spaceship at the other spaceship. His collegue Johnny says "You are an idiot. It is the other spaceship that is really short since it is travelling with respect to us. Your shot has missed." Who is right? Why? What is wrong with the other's argument. (Note, you can assume that the distance between the spaceships passing each other is much less than the length of the spaceships.)

Assuming that the signal from the button to the canon goes at the speed of light, the shot will hit the other spaceship. In Peter's analysis, the signal from the button to the cannon travels at c, while the canon travels upwards to meet

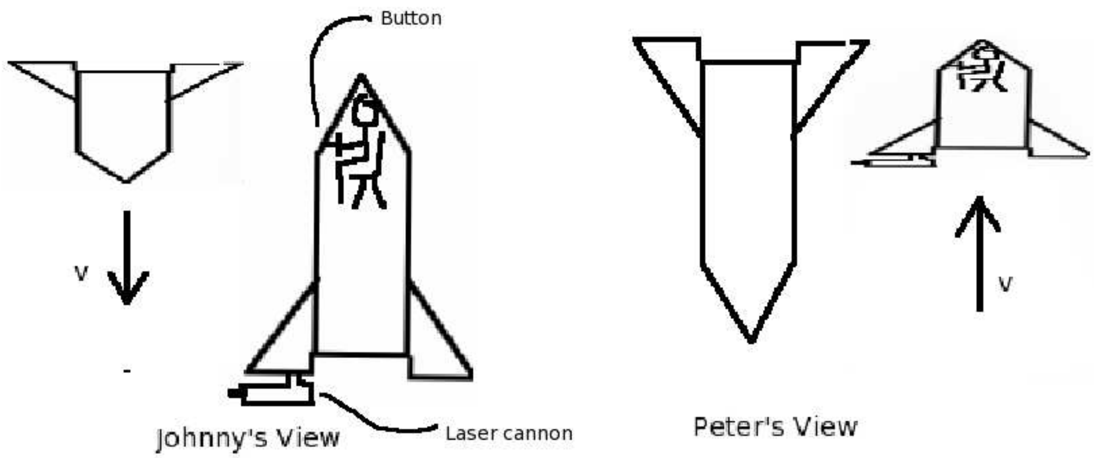

Figure 1: For Problem 4. The two views of the spaceships passing each other. Peter is in the nose of the right spaceship where he presses the button to fire the laser canon in the tail of his spaceship.

the signal. Thus the canon will fire before the tail of the of Peter's spaceship has passed the tail of the otehr ship.

From Johnny's point of view, the signal will again tranvel back to the cannon at the speed of light, while the tail of the spaceship races with that signal at just slightly less than the speed of light (about .9c if the contraction of the figures is to be believed). Thus the other spaceships tail will be just about at the cannon when it fires, into the side of the spaceship.

IF the signal tranvels more slowly than c (and in particular more slowly than about .9c,) the tail of the other spaceship will pass the cannon before the signal gets to the cannon and the shot will miss, but not in the way that Johnny expected.

[Marking– The speed of the signal is a crucial feature here. If they just assume that the speed is c that is OK. If they realise that this is a crucial unstated feature of this problem, give a bonus mark of [1].]

5) A Helium nucleus at rest with rest-mass energy of 4GeV absorbs a gamma ray of energy 1GeV leaving a single nucleus. What is the rest-mass of the resulting nucleus, and what is its velocity?

The easiest way is to use conservation of energy and momentum. FOr a photon the momentum equals the energy. Thus, before the collision, the total energy is 4GeV+1GeV= 5GeV. [1]The momentum is 0GeV+1GeV=1GeV [1] in the direction of the photon. Thus the final nucleus must have energy 5GeV and momentum 1GeV. Its rest mass is thus $\sqrt{5^2 - 1^2}GeV = 4.9GeV$. [1] Its velocity is $v = \frac{p}{E} = \frac{1}{5}$ or $\frac{1}{5}c$[1].

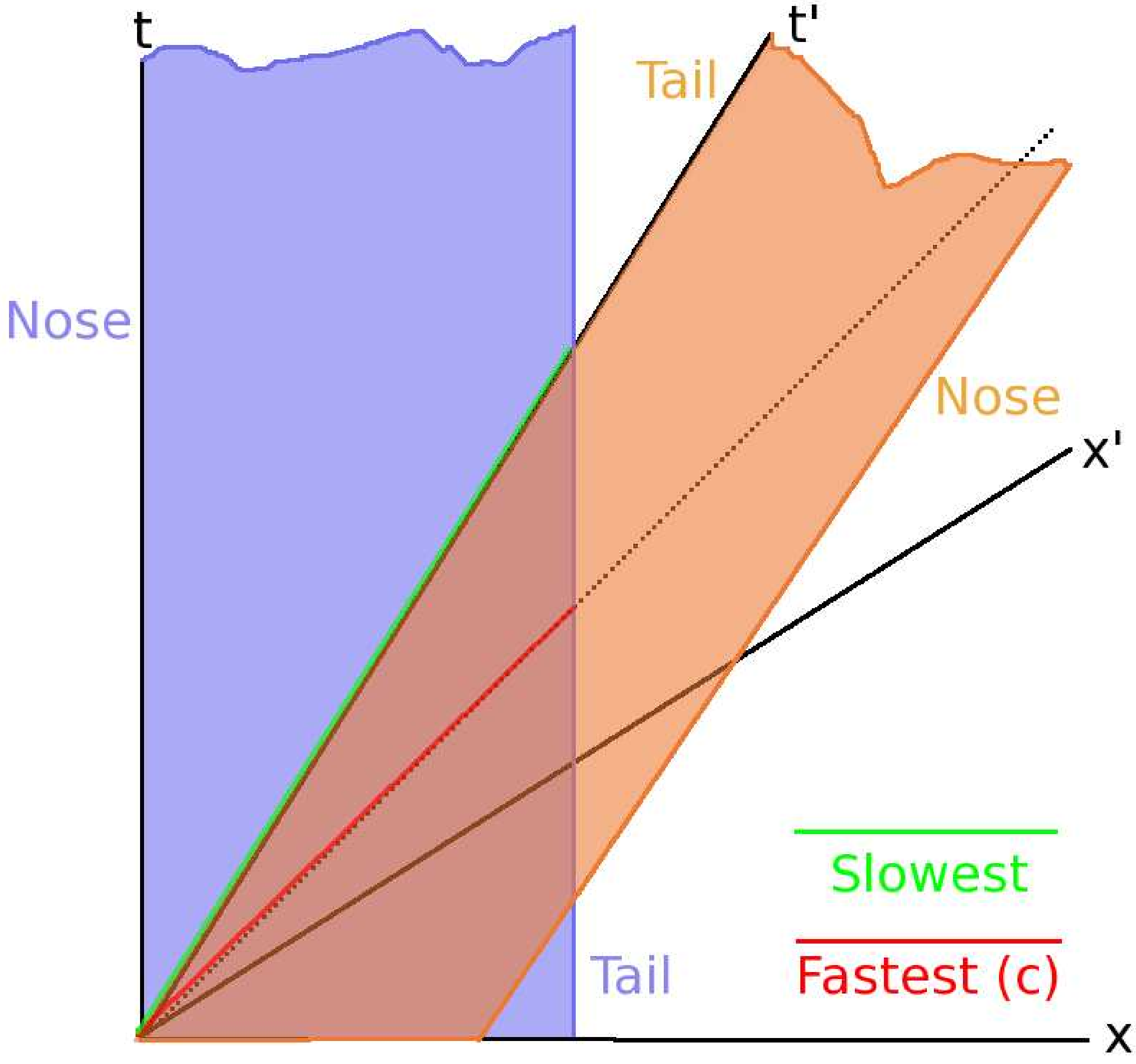

Figure 2: Problem 4: Here is a spacetime diagram of the situation. The blue stripe is the spaceship with the laser at the tail and the button in the nose. At the point when the tail of the other is lined up with the nose/button, the button is pushed. the signal from the button travels down the spaceship with a max velocity of c (the red line). When it reaches the tail of the blue spaceship, the laser fires. Note that the second spaceship (orange) is still at the location of the tail of the first and will get hit. IF the signal travels more slowly– eg at exactly the same velocity as the second spaceship, that signal from the button will just keep up with the tail and the laser would (if we neglect the distance between teh spaceships) just hit it in the tail. If the signal travelled even more slowly, (the path would be more vertical than the green line in this diagram) it would miss. Note that just as John and Peter argue, in the frame of each spaceship (t-x for the laser-button and t'x' for the other) along the line t=const, the orange spaceship is shorter than the blue, while along the time t' constant (x' axis) the blue spacship is shorter than the orange. Ie, both Peter and John are right in their observation of which spaceship is shorter in the different frames, but this is irrelevant as what is important is the speed of the signal from the nose to the laser canon in the tail.