Physics 200-05
Assignment 8

1. Consider the two two-particle states given by

$$|\psi\rangle = N_\psi \left(|+1;1\rangle \otimes |-1;1\rangle + |-1;1\rangle \otimes |+1;1\rangle \right) \tag{1}$$
$$|\phi\rangle = N_\phi \left(|+1;1\rangle \otimes |+1;1\rangle + |-1;1\rangle \otimes |-1;1\rangle \right) \tag{2}$$

where $|+1;1\rangle$ means the eigenstate with the $+1$ eigenvalue for the attribute Σ_1 in the case of the first particle and Ξ_1 for the second. What are possible values for the normalisation factors N_ψ and N_ϕ? What is the inner product $\langle\phi||\psi\rangle$.

(Do not try to expand out the direct products in terms of matrices.)

2. If Σ_1 is the Pauli spin matrix for the first particle with matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and Ξ_1 is the Pauli spin matrix for the second particle with the same matric $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, show in problem 1 that both $|\psi\rangle$ and $|\phi\rangle$ are eigenvectors of the matrix $\Sigma_1 \otimes \Xi_1$. What are the eigenvalues?

3. Show expicitly that if

$$|1;3\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{3}$$
$$|-1;3\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4}$$

for each of the two particles, then the four dimensional vector

$$|1;3\rangle \otimes |1;3\rangle + |-1;3\rangle \otimes |-1;3\rangle \tag{5}$$

cannot be writen as a simple product

$$|\chi\rangle \otimes |\xi\rangle \tag{6}$$

for any choice of $|\chi\rangle$ and $|\xi\rangle$. Such a vector for two particles which cannot be written as a simple product of vectors for the two single particles is called an entangled state.

(In this case do expand the direct product in terms of matrices.)

[Note: in the physics literature, that $\otimes$ symbol is almost always omitted. Thus $|\chi\rangle \otimes |\xi\rangle$ is written as $|\chi\rangle|\xi\rangle$ and you are expected to know that it is the direct product that is being used if you are refering to two separate particles. Similarly $\Sigma_1 \otimes \Xi_1$ is written as $\Sigma_1\Xi_1$ where you are to remember that this is a direct product not a matrix product because Σ_1 and Ξ_1 belong to two separate particles. It is never correct to matrix multiply attributes which belong to separate particles.

4.Show that if I take

$$H = \frac{E}{2}\Sigma_1 \ , \tag{7}$$

in the Heisenberg representation any multiple of Σ_1 does not change in time.

Show that the matrix

$$\Sigma_x = cos(\frac{E}{\hbar}t)\Sigma_2 - sin(\frac{E}{\hbar}t)\Sigma_3 \tag{8}$$

solves the equation of motion for Σ_x.

$$i\hbar\frac{\partial\Sigma_x}{\partial t} = [\Sigma_x, H] \tag{9}$$

(You will need to remember what the value is for the product of two Σ matrices)

5. Show that if $|\psi\rangle$ is a normalised vector which depends on time, that

$$\langle\psi||\dot{\psi}\rangle + \langle\dot{\psi}||\psi\rangle = 0 \tag{10}$$

where the dot denotes a time derivative. By taking the time derivative of

$$\langle a|A|a\rangle = a \tag{11}$$

for an eigenvector $|a\rangle$ show that if A obeys the Heisenberg equations of motion that a does not change in time. (Note that in general $|a\rangle$ will change in time if A does.