Physics 200-05
Assignment 7

1. Consider the state vector

$$|\psi\rangle = \frac{1}{\sqrt{(2)}} \begin{pmatrix} 1 \\ \frac{1+i}{\sqrt{2}} \end{pmatrix} \tag{1}$$

a)What is the unit vector $|\phi\rangle$ orthogonal to this vector? Ie, $\langle\phi||\psi\rangle = 0$?

b) Show that the matrix $A = |\psi\rangle\langle\psi| - |\phi\rangle\langle\phi|$ has eigenvalues ± 1 and eigenvectors $|\psi\rangle$ and $|\phi\rangle$. (Remember that $|\mu\rangle\langle\nu|$ is the product of a column vector times a row matrix, which is a 2x2 matrix if the $|\mu\rangle$ and $|\nu\rangle$ are 1x2 vectors.)

$$\begin{pmatrix} a \\ b \end{pmatrix} \begin{pmatrix} c & d \end{pmatrix} = \begin{pmatrix} ac & ad \\ bc & bd \end{pmatrix} \tag{2}$$

2)Given that the probabilities for rain is
1cm $\rightarrow$ .5
2cm $\rightarrow$ .3
3cm $\rightarrow$ .1
0cm $\rightarrow$ .1

a) What is the expectation value for the rain amount? What is the uncertainty.

b) If there were 30 days on which the above were the forcast, on how many days would expect there to at least 2 cm of rain?

3) Show that

$$[A, BC] = [A, B]C + B[A, C] \tag{3}$$

where $A,\ B,\ C$ are matrices and $[A, B] = AB - BA$ is the commutator.

Show that if X and P obey

$$[X, P] = i\hbar I \tag{4}$$

and if we define the Energy as

$$H = \frac{1}{2m}P^2 + \frac{k}{2}X^2 \tag{5}$$

where m and k are real numbers. Then

$$[X, H] = i\hbar \frac{1}{m} P \tag{6}$$

and

$$[P, H] = -i\hbar k X \tag{7}$$

4) Given

$$|\psi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ \frac{1-i}{\sqrt{2}} \end{pmatrix} \tag{8}$$

find the expectation value and the uncertainty of the attribute represented by the matrix

$$\Sigma_1 + \Sigma_3 \tag{9}$$