Physics 200-04
Assignment 5

1)Old Quantum Mechanics: The Bohr Sommerfeld Quantum rules stated that if the classical orbit was closed (returned on itself) then the quantum rule was that

$$\int p\dot{q}dt = nh \tag{1}$$

where the configuration variable (eg the positions) is q, and p is the momentum. Assuming that the momentum is $p = mv = m\dot{q}$,(ie the dot denotes derivative with respect to time) and we are looking at a harmonic oscillator

$$m\ddot{q} = -kq \tag{2}$$

Recall that the most general solution to this equation is

$$q = A\sin\left(\sqrt{\frac{k}{m}}t + \delta\right) \tag{3}$$

where A and δ are constants. What is the total energy as a function of A, k, m, and δ? Show that the quantum condition lead to the result

$$E_{harm\ osc} = nh\nu \tag{4}$$

where ν is the frequency of the oscillator. (The true quantum answer is $E = (n + \frac{1}{2})h\nu$)

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2a)Consider the earth in circular orbit around the sun. What would the be the difference in radius for two adjacent energy levels of the earth? (Remember that the quantization condition for the earth is that the momentum of the earth times its velocity integrated over time around one orbit is equal to an integer times h. In this case this integer is called the principle quantum number. I want the difference in radii corresponding to adjacent integers where one of the integers corresponds to the current orbit of the earth.) What is the energy difference between these two orbits as a fraction of the earth's orbital energy?

Hint: You need to use the various pieces of information about Newtonian orbits– Kepler's laws, Newton's laws for orbits, etc.

b)Bohr Correspondence: Show that the frequency of the emitted photon or graviton from the earth in circular orbit around the sun from the decay between two adjacent principle quantum numbers for the earth approximately equals the orbital period of the earth. Ie, show from the first part that the energy is proportional to $\frac{K}{n^2}$ where K is a constant expressed in terms of the masses of the earth and sun, and the Newtonian Gravitational constant. Then show that the derivative of E with respect to n ($E_n - E_{n-1} = \frac{E_n - E_{n-1}}{(\Delta n = 1)} \approx \frac{dE_n}{dn}$) divided by h is just the orbital frequency for the current radius of the earth's orbit.

Bohr's correspondence principle basically says that at large quantum numbers the quantum system should behave like a classical system. Since for a classical system one would expect the frequency of the emitted radiation to equal the orbital frequency, the quantum frequency of transition between adjacent levels should approximately be the classical frequency.

3. Calculate the following complex operations: i) $(2-5i)((1-3i)-(1+i))$

ii) $(5+10i)(1+i)/(4-3i)$

iii)Find the roots of the following equation, using complex numbers if necessary

$$x^2 + 2x + 2 = 0 \tag{5}$$

iv) Expand the expression $(x+4i)(x-1+i)/(x-i)$ into a single expression $a + bi$ where both a and b are real numbers. Assume x is a real number.

4. Multiply the matrices

i)

$$\begin{pmatrix} 1+i & 1+i \\ 1-i & 1-i \end{pmatrix} \begin{pmatrix} -1 & i \\ -i & +1 \end{pmatrix} \tag{6}$$

ii) Find the transpose, the Hermetian transpose, and the inverse of the matrix

$$\begin{pmatrix} 1+i & 1+i \\ 1-i & 1+i \end{pmatrix} \tag{7}$$