Physics 200-04
Assignment 4

1) French– Problem 6.5 [Note– in problems from French, the most important thing is to show how you go the answer, not the answer itself.]

[5]

The problem list everything in terms of the velocities, so one must use the expression for the energy and the momentum in terms of the actual velocities (not the proper velocities). The Energy and momentum are

{1}

$$p_t = E = \frac{m}{\sqrt{1-v^2}} \tag{1}$$

$$p_x = \frac{mv_x}{\sqrt{1-v^2}} \tag{2}$$

$$p_y = \frac{mv_y}{\sqrt{1-v^2}} \tag{3}$$

Let us assume that the Mass of the particle before the decay is M and the mass of each particle afterwards is m. Then the conservation equations are

{2}

$$M = \frac{m}{\sqrt{1-v_1^2}} + \frac{m}{\sqrt{1-v_2^2}} + \frac{m}{\sqrt{1-v_3^2}} = m\left(\frac{5}{4} + \frac{5}{3} + \frac{1}{\sqrt{1-v_3^2}}\right) \tag{4}$$

$$0 = \frac{-mv_1}{\sqrt{1-v_1{}^2}} + 0 + \frac{mv_3\cos(\theta)}{\sqrt{1-v_3{}^2}} \tag{5}$$

$$= m\left(\frac{-4}{3} + \frac{v_3\cos(\theta)}{\sqrt{1-v_3{}^2}}\right) \tag{6}$$

$$0 = 0 - \frac{mv_2}{\sqrt{1-v_2^2}} + \frac{mv_3\sin(\theta)}{\sqrt{1-v_3^2}} \tag{7}$$

$$= m\left(-\frac{3}{4} + \frac{v_3\sin(\theta)}{\sqrt{1-v_3^2}}\right) \tag{8}$$

Taking the first term in each of the second and third equations to the right side, squaring and adding, we get

{1}

$$\frac{16}{9} + \frac{9}{16} = \frac{v_3^2}{1-v_3^2} \tag{9}$$

$$v_3 = \sqrt{\frac{16^2+9^2}{9\cdot 16 + 16^2 + 9^2}} = \sqrt{\frac{337}{481}} \tag{10}$$

From the first equation, energy conservation, we have

{1}

$$M = m\left(\frac{5}{3} + \frac{5}{4} + \frac{12}{\sqrt{481}}\right) \tag{11}$$

from which we find m easily.

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2) [Based on French 6-8]. A propulsion system has been proposed where a strong llaser is shone at a totally reflecting "sail" in space. The sail is assumed to be perfectly reflecting in its own rest frame. Ie, the energy of the photon reflected equals the incident energy in this frame (assuming that the rest mass energy of the sail is much greater than that of the photon) Ie, you can assume that in the frame the sail, the photon has the same energy after reflection as when it was incident.]

i) First, assume that the sail is much heavier than the particle. show that if the sail is travelling with velocity v, the energy transfered to the sail by a single photon of incident energy ϵ (travelling in the same direction as the sail) is $2\epsilon\frac{v}{1+v}$. (Hint– transform the photon to the frame moving with the sail, assume specular reflection and then transform the reflected photon back to the original frame.

ii) Consider the photons emitted from the source at n per second. How many photons per unit length are there travelling from the source to the sail? What is the total number between the source and the sail when the sail is a distance x from the source. How many photons per second hit the sail in the frame of the source? What is the energy transfer per unit time to the sail?

[5]

In the rest frame of the sail, the photon energy is assumed not to change (which is true to lowest order in $\left(\frac{\epsilon}{M}\right)^2$ where M is the mass of the sail and ϵ is the energy of the photon. However, the photon's momentum changes from ϵ to $-\epsilon$ {1}(recall that the photon's momentum equals its energy), and thus imparts a momentum of 2ϵ to the sail. Ie, after the collision the momentum of the sail is 2ϵ. If we assume that in the frame of the earth, the sail is travelling at velocity v, then the energy and momentum of the sail before the collision was $E = \frac{M}{\sqrt{1-v^2}}, p = Ev$. after the collision it is

{1}

$$E' = E + 2\epsilon\frac{v}{\sqrt{1-v^2}} \tag{12}$$

$$p = \frac{1}{\sqrt{1-v^2}}\left(Mv + 2\epsilon\right) = E'v + 2\epsilon\sqrt{1-v^2} \tag{13}$$

However this is expressing everything in terms of the energy of the photon in the rest frame of the sail, rather than the frame of the earth in terms of which we are sending out the photons. In the rest frame of the earth, the energy of the photon is

$$\tilde{\epsilon} = \frac{1}{\sqrt{1-v^2}}(\epsilon + v\epsilon) = \sqrt{\frac{1+v}{1-v}}\epsilon \tag{14}$$

Thus in terms of the energy of the sail in the rest frame of the earth the change in energy of the sail in the photon collision is

{1}

$$E' - E = 2\tilde{\epsilon}\frac{v}{1+v} \tag{15}$$

We now have to calculate how many photon collisions there are per second. The photons leave the source at the rate of n per second. The number of photons that are still on their way from the source to the sail are nx where x is the distance to the sail. Thus the number of photon collisions per second is $n - \frac{dnx}{dt} = n(1-v)$ (ie it is the number leaving the source per second minus the number by which the "storage" of photons between the source and the sail increases per second.) Thus the number of photons hitting the sail per second is $n(1-v)$ {1}. Thus the total energy change per second is

{1}

$$\frac{dE}{dt} = 2n(1-v)\tilde{\epsilon}\frac{v}{1+v} \tag{16}$$

To complete the expression, we can evaluate v in terms of E as $v = \frac{p}{E} = \frac{\sqrt{E^2-M^2}}{E}$

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3) French 6.10

[5]

This problem is somewhat ambiguous. The statement that the atom's excited state is Q_0 above the ground state is supposed to mean in the rest frame of the atom. Thus the energy of the moving excited atom is $\frac{m+Q_0}{\sqrt{1-v^2}}$. {1} The rules of energy conservation state that

{1}

$$\frac{m+Q_0}{\sqrt{1-v^2}} = Q + m \tag{17}$$

The statement of momentum conservation is

{1}

$$\frac{m+Q_0}{\sqrt{1-v^2}}v = Q \tag{18}$$

Dividing the first by the second

{1}

$$v = \frac{Q}{Q+m} \tag{19}$$

Also squaring the first and subtracting the second gives

$$(m+Q_0)^2 = (Q+m)^2 - Q^2 \tag{20}$$

Solving for Q we get
{1}

$$Q = \frac{(m+Q_0)^2 - m^2}{2m} \tag{21}$$

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4) French 6.12
[3]

The proton rest mass is 1GeV, so the energy of the first anti-proton is $\frac{5}{3}$ GeV, and of the second is 1GeV. The total energy is thus $\frac{8}{3}$ GeV.

The momentum of that antiproton

$$p^2 = E^2 - m^2 = (\frac{25}{9} - 1)GeV^2 = \frac{16}{9}GeV^2 \tag{22}$$

or $p = \frac{4}{3}$GeV

Thus for the one of the two photons whose direction of travel is the same as the antiproton (the other, I call it the second, travelling in the opposite direction)

$$\epsilon_1 + \epsilon_2 = \frac{8}{3}GeV \tag{23}$$

$$\epsilon_1 - \epsilon_2 = \frac{4}{3}GeV \tag{24}$$

or

$$\epsilon_1 = 2GeV \tag{25}$$

$$\epsilon_2 = \frac{2}{3}GeV \tag{26}$$

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5) French 6.14
[3]– one for each.

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i)After the collision, by the statement of the problem the resultant particle is still an electron, which has the same rest mass as the original electron. Thus the rest mass of the electron does not change. Thus taking the 4-length squared of both sides of the energy momentum equation we have

$$m^2 + 2\epsilon m = m^2 \tag{27}$$

which has solution only if $\epsilon = 0$.

ii)Again taking the length squared of both sides, we have

$$0 = 2m^2 + 2E_1E_2 - \vec{P}_1 \cdot \vec{P}_2 \tag{28}$$

But the magnitude of $\vec{P}_1$ and $\vec{P}_2$ are both less than the values of E_1 and E_2 and thus the last term must be less than the value of the second last term. Ie, the right hand side must be greater than 0.

iii) This is the inverse of the case above, and is impossible for the same reason.

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6) French 7.7a

[2]

We have the Lorentz transformation of the energy and momentum

$$\tilde{p}_x = \frac{1}{\sqrt{1-v^2}}(p_x - vE) \tag{29}$$

Thus in order that this be zero, we must have $v = \frac{p_x}{E}$, But since the total momentum of the photon equals the energy, we have $p_x = -cos(\alpha)E$, so $v = -cos(\alpha)$.

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7) French 7.1

[3]

Evaluating the 4-length of the energy momentum vectors, we have

$$M_K^2 = 2m_\pi^2 + 2m_\pi E_\pi \tag{30}$$

where E_π is the energy of the pion which is not at rest. (since one of the pions has zero momentum, $\bar{p}_1 \cdot \bar{p}_2 = p_{t1}p_{t2} = m_\pi E_\pi$). Thus,

$$E = \frac{M_k^2 - 2m_\pi^2}{2m_\pi} = 753MeV \tag{31}$$