

Physics 200-04
Assignment 3

1. Aberration: Assume that a star is seen to be at an angle of θ with the positive x axis and assume that its location is in the $x - y$ plane. Show that in a frame travelling at velocity v with respect to the first frame along the x axis the angle $\tilde{\theta}$ is given by

$$\tan(\tilde{\theta}) = \frac{\sin(\theta)}{\cos(\theta) + v} \sqrt{1 - v^2} \quad (1)$$

Show that to lowest order in v , this is the same as the expression for the angle as for the non-relativistic aberration angle.

As a spaceship travels at very near the velocity of light, show that the stars all become crowded into the forward direction.

2. Twins Paradox: In class I gave one explanation of the twin's paradox. This will be another one. (It was one that I published in the American Journal of Physics in the early 1980's).

Two twins, Alice and Bob take trips. Alice's trip is to stay at home. Bob travels away at high velocity v , for a time T in Alice's frame, and immediately returns home with the same velocity. They both have telescopes to look at the other. Before they left each carefully measured the other's height (in a direction perpendicular to the direction of travel). Each carefully observes the other during the whole trip. By measuring the angular height of the other as seen through the telescope, each can determine the distance that the other is away from himself by using

$$distance = \frac{H}{\delta\theta} \quad (2)$$

where H is the known height of the other and $\delta\theta$ is the measured height of the other as seen through the telescope. Assuming that the velocity of light is c they can thus calculate how long before he sees the other, the light left the other's position.

a) Show that all along the trip each "sees" (after correcting for the time of travel) the other's clock tick more slowly by $\sqrt{1 - v^2}$.

b) On a graph, plot the path (distance away vs time as corrected for light travel time) that the person sees the other travel. Ie, if

$$X(t) = \frac{H}{\delta\theta(t)} \quad (3)$$

where t is the time at which that angle $\delta\theta$ is measured by that observer, plot $t - X(t)$ vs $X(t)$ for each observer seeing the other.

Note you will probably need to use the aberration formula from problem set 1 where you can assume that θ is very small. (Ie, the height over the distance is small)

In particular see what happens to the height as measured by B at his turnaround of A, and what this means for the distance he ascribes to A and the time that the light left A.

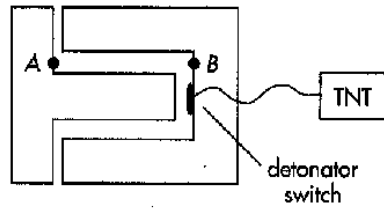
3. Doppler shift: A light flashes once a second (according to its own clock). It is travelling with velocity $.9c$ with respect to an observer, in the direction of the positive x direction and at a distance of 1 light day away along the y direction. What is the frequency of the flashes as seen by the observer as a function of time. What is the limiting frequency when the x value is very large and negative, when $x = 0$ and when x is large and positive.

4. An hydrogen atom of rest mass energy 1Gev emits a photon with energy 10 ev. What is the rest mass of the H atom after the emission? If the photon had an energy of $.5\text{GeV}$, what would the rest mass energy of the H atom be afterward.

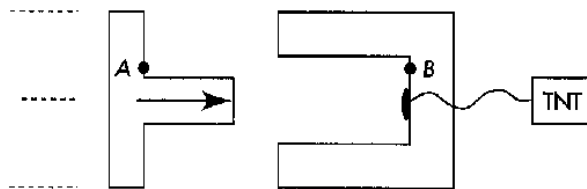
5. This problem is taken from Wheeler and Taylor (6.5)

A structure as in figure 1 is such that the projection of the structure A would just touch the detonator when the flanges of figure A just touch the bars of Figure B. Now A is shot at B at a sizeable fraction of the speed of light. In the frame of figure B, A is shortened and thus the flanges will touch the bars of A before the projection touches the detonator, and will stop A before the projection hits the detonator. In A's frame however, B is shorter, and the projection of A hits the detonator before the flanges touch B and the bomb is detonated.

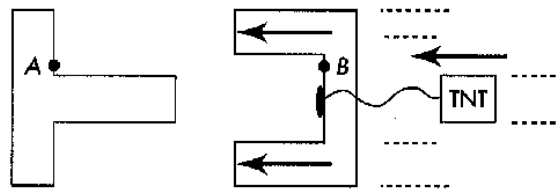
Is the bomb detonated or not? What is wrong with the argument that predicts the opposite?



BOTH AT REST



REST FRAME OF U STRUCTURE



REST FRAME OF T STRUCTURE

EXERCISE 6-5. *Both at rest:* The leg of the T almost reaches the detonator switch when both the T and the U are at rest. Points A and B are used in part b of the exercise. *Rest frame of U structure:* The leg of the moving T is Lorentz contracted in the rest frame of the U. Does this mean that the explosion will not take place? *Rest frame of T structure:* The legs of the moving U are Lorentz-contracted in the rest frame of the T. Does this mean explosion will take place?