

Physics 200-05  
Assignment 2

1. Given the two matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 1 \\ 3 & 1 & 0 \end{pmatrix} \quad (1)$$

$$B = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

Find the matrices  $AB$ ,  $BA$ ,  $A^T$ ,  $B^T$ , and  $B^{-1}$ . Does the matrix  $A$  represent a rotation? Does the matrix  $B$ ?

(Recall that a rotation is defined by the requirement that  $A^T I A = I$ . I.e, the identity matrix plays the roll of the metric  $G$ .)

2. i) Show that

$$\sinh(\theta) \cosh(\theta') + \cosh(\theta) \sinh(\theta') = \sinh(\theta + \theta') \quad (3)$$

$$\cosh(\theta) \cosh(\theta') + \sinh(\theta) \sinh(\theta') = \cosh(\theta + \theta') \quad (4)$$

Note the similarities **and** differences with the trigonometric formulas you are (I hope) more familiar with.

ii) Using the above formula show that two successive Lorentz transformations both along the  $x$  direction are such that if the velocity of transformation from the first to the second frame is  $v_1$  and of the second to the third frame is  $v_2$  then the velocity from the first to third frame is

$$\frac{v_f}{c} = \frac{\frac{v_1}{c} + \frac{v_2}{c}}{1 + \frac{v_1 v_2}{c^2}} \quad (5)$$

(Use the fact that  $\tanh(\theta) = \frac{v}{c}$ ). I.e, while the rapidities ( the "angle" in the hyperbolic function representation of the Lorentz transformations) of successive Lorentz transformations add, the velocities do not.

3.) Do a Lorentz transformation in the  $x$  direction with  $v_x = c/\sqrt{3}$ . Now in the new frame do a Lorentz transformation in the  $y$  direction with the velocity  $v_y = c/\sqrt{2}$ . Show that the combined transformation is equivalent to a Lorentz transformation along the direction at  $45^\circ$  between the  $x$  and  $y$  axes with a velocity of  $\sqrt{2/3}c$  followed by a rotation around the  $z$  axis.

While the full set of Lorentz transformations form a group, the boosts (which are the Lorentz transformations which leave two of the space-like directions the same) do not. This observation eventually leads to what is called the Thomas

Precession, and is one of the purely kinematic terms which governs the precession of the spins of electrons in an atom. Two boosts along non-parallel directions is equivalent to a Lorentz boost plus a rotation is a generic feature of Lorentz transformations.

4.) Show the consistency of the special relativistic formulas. Consider the following synchronization of clocks. Alice and her friend Amy get together at the origin and synchronize their clocks to each other, ensuring that both show exactly the same time and run at the same rate. Now Amy very slowly (with a velocity  $\delta v$  approaching zero) moves away from Alice to a location  $X$  along Alice's  $x$  axis. Show that in Alice's frame, the time on Amy's clock at  $X$  will be synchronized with her clock. (Show that in the limit as  $\delta v$  goes to zero, the time difference between Amy's time to get to the location  $X$  and Alice's time for Amy to get to  $X$  are the same.) Now let us look at this process from Bob's point of view, who is moving with velocity  $v$  with respect to Alice. Show that Bob will calculate the difference between Amy's time to get to  $X$  and Alice's time is  $\frac{vX}{c^2\sqrt{1-\frac{v^2}{c^2}}}$ .

Hint, use the expression for the rate of Alice's and Amy's clocks according to Bob (time dilation) and look at the difference to lowest order in  $\delta v$ . Find the time it takes Amy to get to the point  $X$  and express the time difference in terms of  $X$ .